

551.468

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G?=>JHKL:LBQKDH?
;HНННН -;HDEBHA:BFHCK LB?
HHEBKHHVNHF

Dexqu_ kehZ

$$\varepsilon = H/L, \quad H - L -$$

ε

$$\varepsilon^2,$$

$$N = (-g\rho_z/\rho_0)^{1/2}, \quad g -$$

ρ

ρ_0

$$\frac{U^2}{L^2 N^2} \ll 1 \quad \Gamma = \varepsilon^2 / \text{Ri} \ll 1, \quad (1)$$

$$\text{Ri} = N^2 H^2 / U^2 - [1] \cdot H = O(10^2) N, \approx 10^{-4}^{-1}, U \approx 0.5 /$$

1 ,

$$w \sim uh_x = utg\alpha ;$$

(1)

2-D-

. - 6 [] 1;
[7- 9] .

σ -

ihklZghdZaZZqb
Mgby dZhuo dhhjbgW

$X \circ Y$

0Z

$$Q_T = Q \times [0, T^*], \quad Q :- \zeta(x, y; t), \quad h(x, y)$$

$$\partial Q, Q = \{x, y, z; x, y \in \Omega, -h \leq z \leq \zeta\}, 0 \leq t \leq T^*,$$

$$\frac{d\mathbf{u}}{dt} + R\nabla p + 2\tilde{\Omega} \times \mathbf{u} = \mathbf{g} + \nabla_2(K\nabla_2 \mathbf{u}) + (\mathbf{v}\mathbf{u}_z)_z, \quad (2)$$

$$\operatorname{div} \mathbf{u} = 0, \quad (3)$$

$$\frac{d\Theta_i}{dt} = \frac{\partial}{\partial z} v_{\Theta_i} \frac{\partial \Theta_i}{\partial z} + \nabla(K_{\Theta_i} \nabla \Theta_i), \quad (4)$$

$$\rho(x, y, z; t) = \rho(\Theta_i), \quad (5)$$

$$d/dt = \partial/\partial t + \mathbf{u} \cdot \nabla; \mathbf{u} = (u, v, w) - \nabla = (\nabla_2, \partial/\partial z); \nabla_2 := (\partial/\partial x, \partial/\partial y) -$$

$$\left(\frac{1}{\rho_0}, \frac{1}{\rho_0}, \frac{1}{\rho} \right); \rho - \rho_0 - ; p - \mathbf{g} = (0, 0, -g) -$$

$$2\tilde{\Omega} \times \mathbf{u} = (f_\Gamma w - fv, fu, -f_\Gamma u), f_\Gamma = 2\tilde{\Omega} \cos \varphi - f = 2\Omega \sin \varphi$$

$$K, v; - \tilde{\Omega} - \varphi -$$

$$K_{\Theta_i}, v_{\Theta_i} - i = 1, 2; \Theta_1 - \Theta_2 - , ,$$

$$- (5) \quad (2) \quad \mathbf{u}, p, \rho, \Theta_i -$$

$$v, v_{\Theta_i}, K, K_{\Theta_i} -$$

$$\zeta . -$$

$$\frac{\partial p}{\partial z} = -g\rho \quad (6)$$

$$p_\Gamma \cdot \rho = \rho_0 + \rho', \rho' \ll \rho_0 \quad (6)$$

$$p|_{z=\zeta} - p_\Gamma = -g\rho_0(\zeta - z) - g \int_z^\zeta \rho' dz. \quad (7)$$

$$p|_\zeta = \text{const} . \quad (7)$$

$$\nabla_2 p_\Gamma = g\rho_0 \nabla_2 \zeta + g \nabla_2 \int_z^\zeta \rho' dz. \quad (8)$$

$$\gamma = \frac{1}{\rho} p_z + g . \quad O(\rho'^2) :$$

$$\frac{1}{\rho} p_z = \frac{1}{\rho_0} (1 - \rho'/\rho_0) p_z = \frac{1}{\rho_0} p_z + g \rho'/\rho_0; \quad \gamma = \frac{1}{\rho_0} p_z + g \rho/\rho_0.$$

$$p = p_\Gamma + p_D, \quad p_D =$$

$$\gamma = \frac{1}{\rho_0} \frac{\partial}{\partial z} (p_\Gamma + p_D) - \frac{1}{\rho_0} \frac{\partial}{\partial z} p_\Gamma = \frac{1}{\rho_0} \frac{\partial p_D}{\partial z}.$$

(2)

$$\frac{\partial}{\partial t} \mathbf{u} + R \nabla \Pi = \underline{\varphi}, \tag{9}$$

$$\Pi = (p, p, p_D), \quad \underline{\varphi} = -(\mathbf{u} \cdot \nabla) \mathbf{u} - 2\tilde{\Omega} \times \mathbf{u} + \nabla_2 (K \nabla_2 \mathbf{u}) + (\mathbf{v} \mathbf{u}_z)_z.$$

$$DjZ\zeta y \ aZZqZ \ \backslash \ djb\hbar ebgcguo \ dhhj\hbar gZlZo$$

$$\xi = \xi(x, y), \quad \eta = \eta(x, y), \quad \sigma = H^{-1}(z - \zeta), \quad t' = t, \tag{10}$$

$$H = h + \zeta$$

$$J^{-1} = \partial(\xi, \eta, \sigma) / \partial(x, y, z), \quad 0 \neq J^{-1} < \infty,$$

$$J = J_* H, \quad J_* = \partial(x, y) / \partial(\xi, \eta) -$$

Q

Q*

Q

∂Q₁,

∂Q₁*

∂Q,

∂Q₂*

Q*

Ω*

$$\sigma = -1 \quad \sigma = 0$$

$$\nabla_2 p_\Gamma = g \rho_0 \nabla_2 \zeta + g \nabla_2 I, \quad I = H \int_{\sigma}^0 \rho' d\sigma. \tag{11}$$

(-9)

(1 0)

$$\frac{\partial}{\partial t} \mathbf{u} + R \nabla \Pi = \underline{\varphi}$$

$$\underline{\varphi} = (\varphi_u, \varphi_v, \varphi_w) = -U^i \frac{\partial \mathbf{u}}{\partial \xi^i} - W \frac{\partial \mathbf{u}}{\partial \sigma} - 2\tilde{\Omega} \times \mathbf{u} + H^{-2} (\mathbf{v} \mathbf{u}_\sigma)_\sigma + J_*^{-1} (K J_* g^{ik} \mathbf{u}_{\xi^k})_{\xi^i}. \tag{12}$$

$$U^i = \mathbf{v} \nabla \xi^i -$$

$$\nabla \xi^i = \mathbf{e}^i = (\xi_x^i, \xi_y^i) -$$

$$i, k = 1, 2$$

$$U^1 = U, \quad U^2 = V, \quad \xi^1 = \xi, \quad \xi^2 = \eta; \quad W = \sigma_t + \mathbf{v} \nabla_2 \sigma + w \sigma_z -$$

$$\nabla_2 = \mathbf{e}^i \partial / \partial \xi^i, \quad g^{ik} = \mathbf{e}^i \mathbf{e}^k -$$

:

$$\frac{\partial J}{\partial t} + \frac{\partial}{\partial \xi^i} J U^i + \frac{\partial}{\partial \sigma} J W = 0 \quad (13)$$

$$\frac{\partial}{\partial \xi^i} J U^i + \frac{\partial}{\partial \sigma} J \hat{W} = 0, \quad (13)$$

$$\hat{W} = W - \sigma_t .$$

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\rho_0} \nabla_2 p_\Gamma = \underline{\Phi}_v . \quad (14)$$

$$\underline{\Phi}_v = (\Phi_u, \Phi_v) .$$

$\Theta :$

$$\frac{\partial \Theta}{\partial t} + U^i \frac{\partial}{\partial \xi^i} \Theta + W \frac{\partial}{\partial \sigma} \Theta = \hat{D}(\Theta) . \quad (15)$$

$$\hat{D} = K_\Theta J_*^{-1} \left[\frac{\partial}{\partial \xi} \left(J_* g^{11} \frac{\partial}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left(J_* g^{22} \frac{\partial}{\partial \eta} \right) \right] + \frac{\partial}{\partial \sigma} \left(v_\Theta g^{33} \frac{\partial}{\partial \sigma} \right) . \quad (15)$$

(1 2) , - (1 3)
v, v_Θ .

, [1 0] .

∂Q₁^{*} -

∂Q₂^{*} -

Q^{*} ξ = const ,

η = const .

$$U|_{\partial Q_1^*} = 0 . \quad (16)$$

∂Q₂^{*} -

V -

η = const :

$$\frac{\partial V}{\partial n} \Big|_{\partial Q_2^*} = \frac{1}{\sqrt{g^{22}}} g^{2j} V_{\xi^j} , \quad (17)$$

j = 1 , 2 , 3 .

$$U|_{\partial Q_2^*} = \psi_1(\xi, \sigma; t), \quad V|_{\partial Q_2^*} = \psi_2(\xi, \sigma; t) \quad (18)$$

K ∂V/∂n .

ε^{*} τ/ρ₀ :

$$U^i|_{\sigma=-1+\varepsilon^*} = 0, \quad v U_\sigma^i|_{\sigma=0} = H \nabla \xi^i \frac{\tau}{\rho_0} . \quad (19)$$

$$W \quad W|_{-1} = W|_0 = 0. \\ \Theta = (T, S) \quad (1 \ 5)$$

$$g^{1j} \Theta_{\xi^j} \Big|_{\partial Q_1^*} = 0. \quad (20)$$

$$g^{2j} \Theta_{\xi^j} \Big|_{\partial Q_2^*} = 0, \quad (21)$$

$$\Theta \Big|_{\partial Q_2^*} = \Psi_3(\xi, \sigma; t). \quad (22)$$

$$\frac{\partial \Theta}{\partial n} \Big|_{\sigma=-1} = K_{\Theta} g^{3i} \Theta_{\xi^i} + v_{\Theta} g^{33} \Theta_{\sigma}. \quad (23)$$

$$\Theta_1 = T \quad \Theta_2 = S$$

$$\mathbf{u} \Big|_{t=0} = \mathbf{u}^0$$

$$\Theta_i \Big|_{t=0} = \Theta_i^0.$$

$$\rho_0 \mathbf{J} \mathbf{u}, \quad (9)$$

$$\rho_0 (g \zeta + |\mathbf{u}|^2 / 2) \quad) - \quad g I \in \mathcal{A}_D \mathfrak{z}$$

$$e_t + [(A+B)JU]_{\xi} + [(A+B)JV]_{\eta} + [J(AW + B\hat{W})]_{\sigma} + \chi = D, \quad (24)$$

$$e = \frac{1}{2} \rho_0 J_* (H|\mathbf{u}|^2 + g\zeta^2) - \quad A = \rho_0 \left(\frac{1}{2} |\mathbf{u}|^2 + g\zeta \right) -$$

$$B = gI + p_D -$$

$$\chi = Jgwp' -$$

$$D = \rho_0 H \mathbf{u} \left(\frac{\partial}{\partial \xi^i} K J_* g^{ik} \frac{\partial \mathbf{u}}{\partial \xi^k} + J_* H^{-2} \frac{\partial}{\partial \sigma} v \frac{\partial \mathbf{u}}{\partial \sigma} \right). \\ Q^* \quad (2 \ 4)$$

$$U \Big|_{\partial Q_1^*} = 0, W \Big|_{\sigma=-1} = W \Big|_{\sigma=0} = 0, \hat{W} \Big|_{\sigma=-1} = 0, I \Big|_{\sigma=0} = 0, p_D \Big|_{\sigma=0} = 0$$

$$E_t + \int_{\partial Q_2^*} (A+B)JV d\xi d\eta + \int_{Q^*} gwp' J dQ^* = D_1 + D_2 + D_3. \quad (25)$$

$$E = \int_{Q^*} edQ^* -$$

$$D_1 = -\rho_0 \int_{Q^*} K g^{ik} \frac{\partial H \mathbf{u}}{\partial \xi^i} \frac{\partial \mathbf{u}}{\partial \xi^k} + H^{-1} \nu \left(\frac{\partial \mathbf{u}}{\partial \sigma} \right)^2 J_* dQ^* \quad -$$

$$D_2 = \int_{S^\zeta} \mathbf{u} \cdot \boldsymbol{\tau} J_* d\xi d\eta - \int_{S^h} \mathbf{u} \cdot \boldsymbol{\tau}_h J_* d\xi d\eta \quad -$$

$$\boldsymbol{\tau} = \rho_0 H^{-1} \nu \frac{\partial \mathbf{u}}{\partial \sigma} \quad S^\zeta \quad S^h;$$

$$D_3 = \frac{1}{2} \rho_0 \int_{\partial Q_*^*} HK \left(g^{12} \frac{\partial}{\partial \xi} |\mathbf{u}|^2 + g^{22} \frac{\partial}{\partial \eta} |\mathbf{u}|^2 \right) J_* d\xi d\eta \quad -$$

(1 4)

$$\tilde{w}_t = \varphi_w. \quad (27)$$

$$(1 4) , \quad \mathbf{u}_1 = \mathbf{u}, \tilde{w} , \quad (k+1)\tau, \quad k=0,1,\dots, \quad \bar{k} = \lceil \tilde{T}/\tau \rceil$$

(1 2) :

$$\mathbf{u}_\Gamma^{k+1} = \mathbf{u}^*, \quad (28)$$

$$\begin{aligned} \frac{(\mathbf{v}^* - \mathbf{v}^k)}{\tau} + \frac{1}{\rho_0} \nabla_2 p_\Gamma^* &= \underline{\Phi}_v^*, \\ \frac{(w^* - w^k)}{\tau} &= \Phi_w^*. \end{aligned} \quad (29)$$

$$(1 3) \quad - (23) - (1 6) \quad -$$

$$[1 0] .$$

(2 9)

$$\begin{aligned} \frac{(\mathbf{v}^{k+1} - \mathbf{v}^k)}{\tau} + \frac{1}{\rho_0} \nabla_2 (p_\Gamma^* + p_D^{k+1}) &= \underline{\Phi}_v^*, \\ \frac{(w^{k+1} - w^k)}{\tau} + \frac{1}{\rho_0} \frac{\partial}{\partial \sigma} p_D^{k+1} &= \Phi_w^*, \end{aligned}$$

$$\frac{(\mathbf{u}^{k+1} - \mathbf{u}^*)}{\tau} + \frac{1}{\rho_0} \nabla p_D^{k+1} = 0, \quad (30)$$

$$p_D / \rho_0 = p \quad \delta$$

$$\begin{aligned} \frac{(u^{k+1} - u^*)}{\tau} + p_{\xi^i}^{k+1} \xi_x^i + p_{\sigma}^{k+1} \sigma_x &= 0, \\ \frac{(v^{k+1} - v^*)}{\tau} + p_{\xi^i}^{k+1} \xi_y^i + p_{\sigma}^{k+1} \sigma_y &= 0, \\ \frac{(w^{k+1} - w^*)}{\tau} + p_{\sigma}^{k+1} \sigma_z &= 0. \end{aligned}$$

$$\begin{aligned} \xi_x, \quad - \xi_y, \quad -; \\ \eta_x, \quad , \quad \eta_y; \quad , \\ \sigma_x, \quad , \quad , \quad \sigma_y, \quad - \end{aligned}$$

σ_z . :

$$\begin{aligned} \frac{(U^{k+1} - U^*)}{\tau} + g^{1j} p_{\xi^j}^{k+1} + g^{13} p_{\sigma}^{k+1} &= 0, \\ \frac{(V^{k+1} - V^*)}{\tau} + g^{2j} p_{\xi^j}^{k+1} + g^{23} p_{\sigma}^{k+1} &= 0, \\ \frac{(\hat{W}^{k+1} - \hat{W}^*)}{\tau} + g^{3j} p_{\xi^j}^{k+1} + g^{33} p_{\sigma}^{k+1} &= 0. \end{aligned} \tag{31}$$

$$\eta, \quad - \sigma, \quad J, \quad \xi, \quad -$$

$$(Jg^{ij} p_{\xi^i}^{k+1})_{\xi^j} = [(JU^*)_{\xi} + (JV^*)_{\eta} + (J\hat{W}^*)_{\sigma}] / \tau, \tag{32}$$

$$\begin{aligned} \text{div } \mathbf{u}^{k+1} &= 0, \\ (\xi, \eta), \end{aligned}$$

$$\frac{\partial p}{\partial n} \Big|_{\sigma=-1} = 0, \quad p|_{\sigma=0} = 0; \tag{33}$$

$$\begin{aligned} Q^* \\ \frac{\partial p}{\partial n} \Big|_{\partial Q_1^*} = 0; \end{aligned} \tag{34}$$

$$\frac{\partial p}{\partial n} \Big|_{\partial Q_2^*} = \psi(\xi, \sigma, t), \tag{35}$$

$$\psi^{k+1} = -(V^{k+1} - V^*) / \tau \sqrt{g^{22}}$$

$$s: \psi^{k+1,s} = -(V^{k+1,s} - V^*) / \tau \sqrt{g^{22}}, \quad s = 1, 2, 3, \dots$$

$$\begin{aligned} - (35) \quad (32) \\ (31) \quad \mathbf{U}^{k+1} = (U, V, \hat{W})^{k+1}, \end{aligned}$$

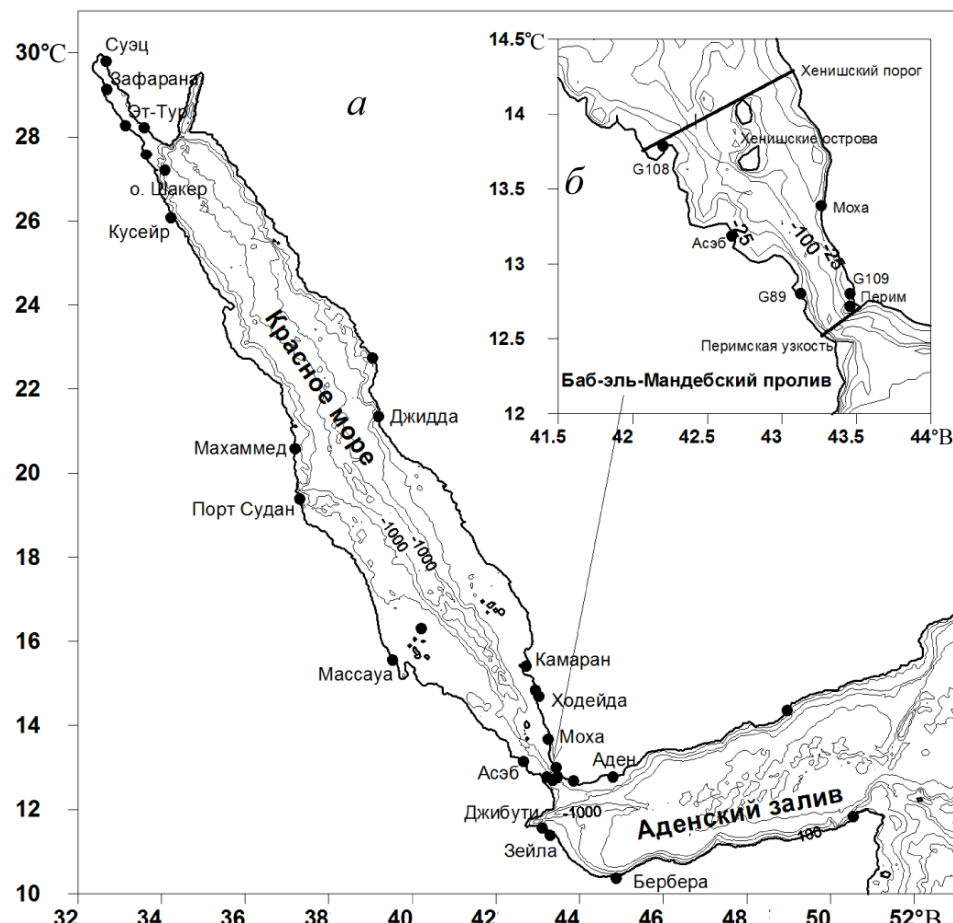
$$w = H(\hat{W} - u\sigma_x - v\sigma_y); \quad \zeta$$

$$u = J_*(U\eta_y - V\xi_y), \quad v = J_*(V\xi_x - U\eta_x),$$

$$(13), \quad (14).$$

110 1 3 7 6

2 ~ 1



1 а) б).

[1 1 , 1 2] ,

<uqbkeblevgu_ iZjZflju b hpgdZ lhqghklb jrgby
g]bfhklZlbqkdhc aZZqb

33×53

4 0

33×103×40

$$\Delta_{\min} = 110 , \Delta_{\max} = 3500$$

(2) ; .

$$\tau = 90$$

3. .

JamevlZlu

, 4, 5 ,

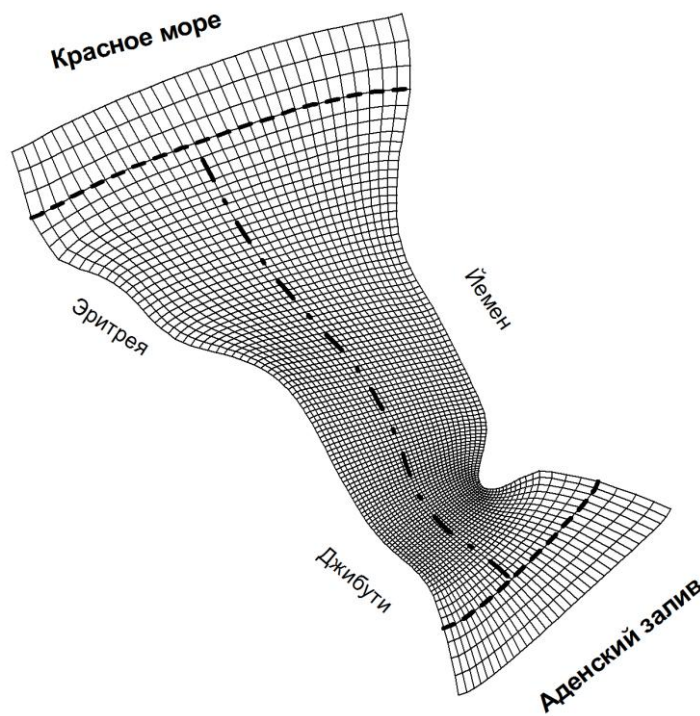
(1)

$$\|\delta(\chi)\|_C = \max_S |\delta\chi| ,$$

$$\delta\chi = \chi_{\text{НГ}} - \chi_{\Gamma} -$$

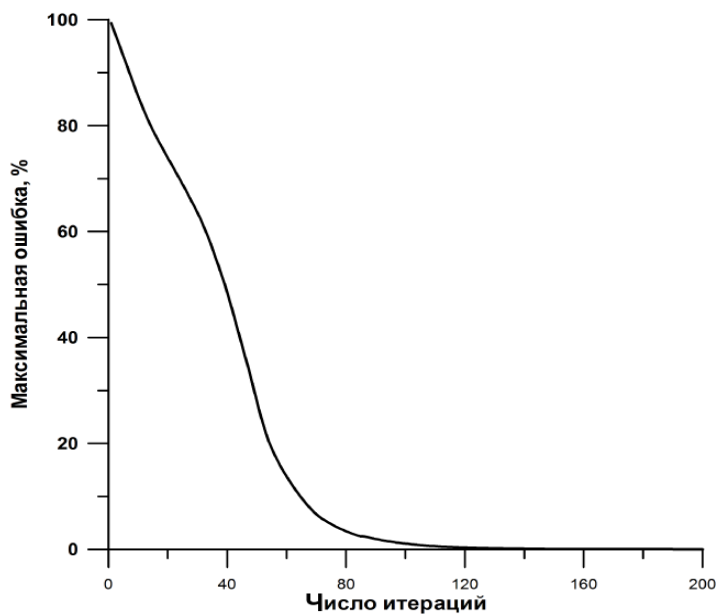
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S -

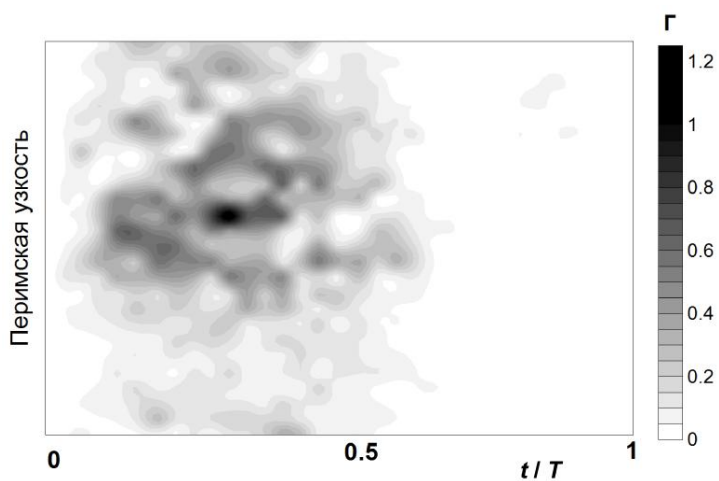


. 2 .

33×103)

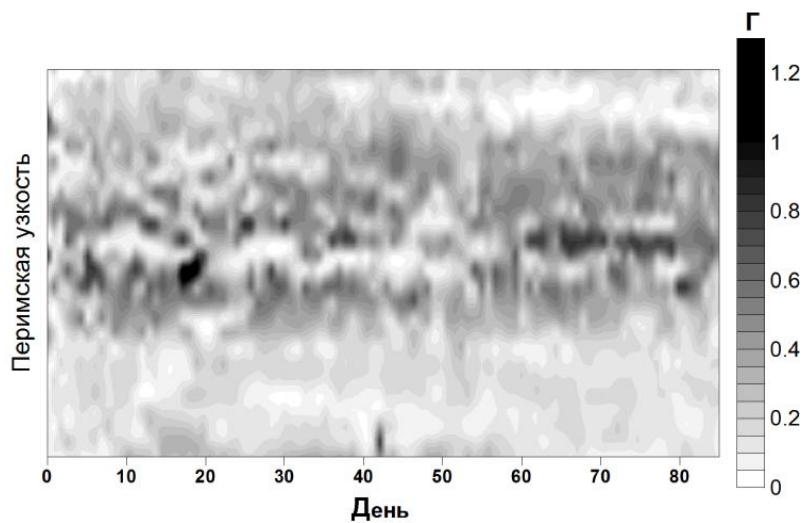


. 3 .



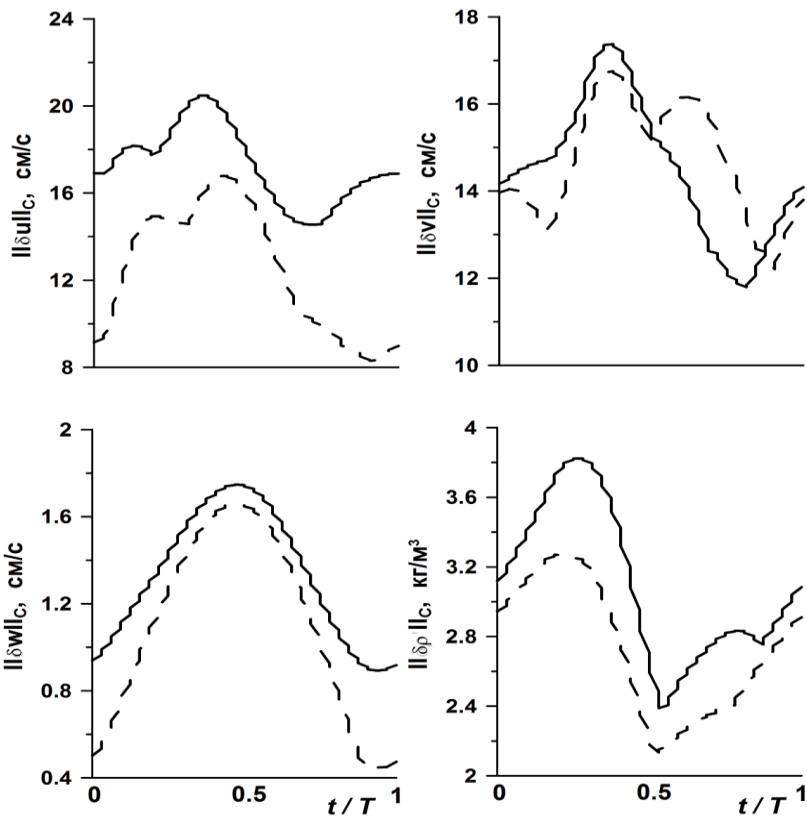
. 4 .

M_2 .



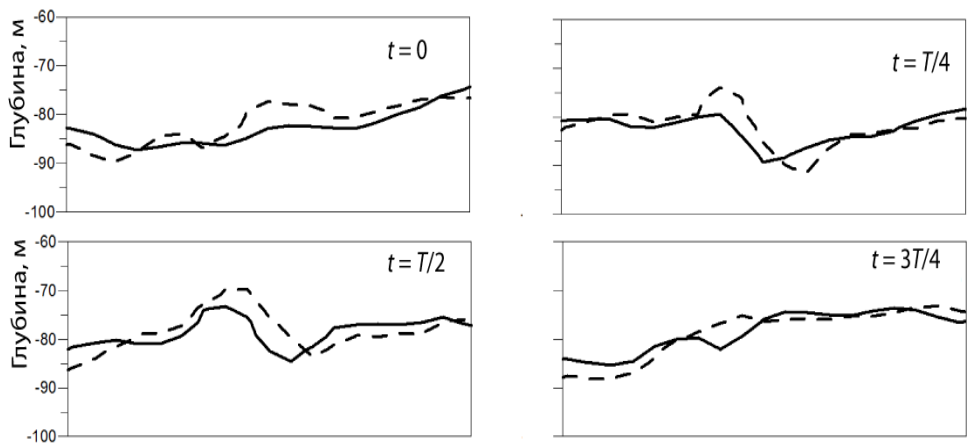
. 5 .

6 . $\chi = (\mathbf{u}, \rho')$ -
 M_2 ,
 χ , $T/2; T -$ % M_2 , 2 0
 ρ' , -
 M_2 , -
 7).
 $T/4$
 1 0 ,



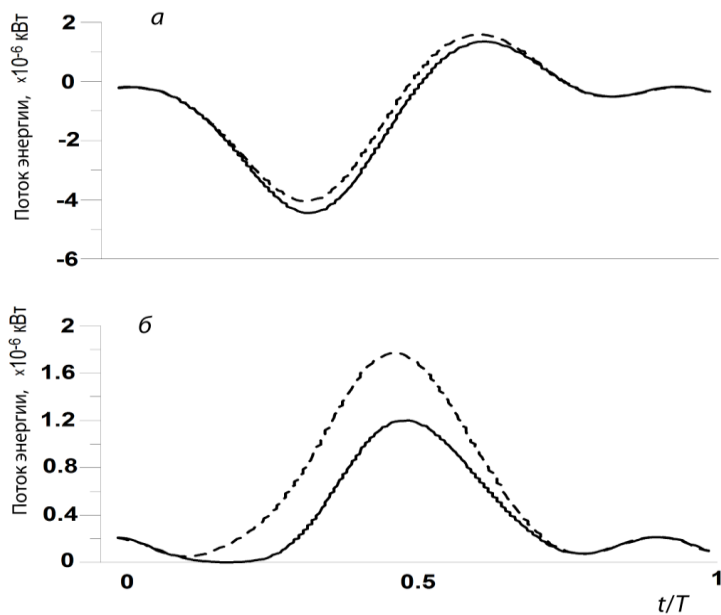
6 . $\|\mu(\chi)\|_C, \mu = \{\mathbf{u}, \rho'\}$

- 33×103 , - 33×53 .



. 7 .

M_2



. 8 .

$a -$ $b -$, ; - - ;

8 . . , 6 4 . ' 1 0 - 2

;

M_2 .

$$\varepsilon = H/L \ll 1 .$$

$$H/L$$

$$(1): \Gamma = \varepsilon^2 U^2 / N^2 H^2 \ll 1 .$$

$$\Gamma$$

$$\varepsilon = H/L = \operatorname{tg} \alpha ; ,$$

$$\Gamma$$

$$\varepsilon$$

$$h - , ; h_0 -$$

$$h_*(x, y) .$$

$$h_0 \leq h_* \leq h .$$

$$(1)$$

$$\gamma = \frac{U^2}{N^2 h_*^2} \operatorname{tg}^2 \alpha .$$

$$\gamma$$

$$(1)$$

$$M_2$$

$$-20 \%$$

EbljZlmjZ

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