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РАСПРЕДЕЛЕНИЕ НАКЛОНОВ ПОДВОДНЫХ СКЛОНОВ В СВЯЗИ С КРИТИЧЕСКИМ ОТРАЖЕНИЕМ ВНУТРЕННИХ ВОЛН

Рассматривается распределение наклонов батиметрии в океане в трехмерной постановке. Рассчитаны крутизна наклонов и горизонтальная ориентация тангенциальных плоскостей. Рассматривается часть Атлантического океана, в которую включены инерционные широты суточных приливов. Выражение для критического отражения внутренних волн переписано с включением нетрадиционного эффекта Кориолиса. Показано, что вероятность критического отражения существенно увеличивается, если в наклонах дна учтена трехмерность по сравнению с расчетами по одной линии.

Ключевые слова: внутренние волны, критические отражения, трехмерность рельефа дна, нетрадиционный эффект Кориолиса.

Internal waves in continuously stratified layers propagate in a beam-like manner, the angle of which is dictated by the waves's frequency ω, the latitude ϕ and the local stratification N [1]. If they impinge on the ocean floor, they reflect from the bottom slope, preserving their angle with the vertical. If the angle of energy propagation (i.e. group velocity) matches that of the bottom slope, one speaks of critical reflection. The reflected beam becomes then much narrower, hence more intense. Such intensification has been shown to occur in the ocean [2]. Presumably, this is accompanied by wave breaking, although no direct observational evidence has been reported on this. But how wide-spread is this phenomenon of critical reflection? In the internal-wave spectra, the low-frequency part predominates, which corresponds to small angles with the horizontal for energy propagation. It was investigated by [3] what the likelihood is of a match with bottom slopes. In particular, so-called non-traditional effects were taken into account; they are due to the (often neglected) horizontal component of the Earth's rotation vector. These effects become important in weakly stratified layers, such as in the deep ocean (for a review of phenomena affected by the horizontal component, see [4].

Ref.[3] concluded that probabilities increase towards inertial latitudes, i.e. latitudes at which the wave's frequency matches the local "traditional" Coriolis parameter $f = 2\Omega \sin \phi$ (Ω being the Earth's angular velocity). The effect of including the non-traditional Coriolis component $\tilde{f} = 2\Omega \cos \phi$ was shown to be dual: it increases the probability below the inertial latitude, but in the vicinity of the inertial latitude it decreases the probability.

However, one point of the analysis in [3] needs further elaboration, namely the distribution of the steepness of bottom slopes. The distribution was based on just four sections (derived from a 1/30° topographic database, [5]). By using latitudinal or longitudinal sections, the three-dimensional bathymetry was treated as if it were uniform in the other horizontal direction. One can argue *a priori* that this approach must produce a bias towards weak slopes: in a section, every time one crosses a maximum or minimum, the steepness is found to be zero, but in the full three-dimensional case a zero steepness is found only if it concurs with one in the other horizontal direction, making the probability lower.

It is the purpose of this paper to straighten out the previous weakness and to show how this changes the results.

Bathymetry and slope-distribution. By way of example, we will consider a slice of the Atlantic Ocean, see Fig.1. The area covers the continental slopes on either side of the basin, but the dominant feature is the Mid-Atlantic Ridge, as is clear from the steepness shown in Fig.1, *b*.

That steepness is calculated as follows. Let the bathymetry be given by $z = h(x, y)$, so that we can define

$$f(x, y, z) = z - h(x, y) = 0 .$$

The gradient is $\nabla f = (-\partial h/\partial x, -\partial h/\partial y, 1)$, which is the vector normal to the surface $f(x, y, z) = 0$, and hence normal to the bathymetry. From this gradient, we can infer three properties. First, its length $L = [(\partial h/\partial x)^2 + (\partial h/\partial y)^2 + 1]^{1/2}$. Second, if we take the inner product with the vertical unit vector (0, 0, 1), we get $\cos(\theta) = 1/L$.

Here θ is the angle between the gradient and the vertical, which is identical to the angle between the tangential plane and the horizontal.

Hence, the steepness γ, the tangent of that angle, is given by

$$\gamma = [L^2 - 1]^{1/2} . \quad (1)$$

Finally, we can calculate the horizontal orientation of the tangential plane, ν, as the angle between the zonal direction and the vector $(\partial h/\partial x, \partial h/\partial y)$. This angle is shown in Fig.1, *c*. It follows predominantly the orientation of the Mid-Atlantic Ridge and the fracture zones perpendicular to it.

We can now calculate the distribution of slopes for the entire area shown in Fig.1, see Fig.2. For comparison, we plot the typical distribution derived by [3] for longitudinal or latitudinal sections (black line). As expected, the genuinely three-dimensional approach gives a smaller probability for the mildest slopes (i.e., lower than about 0.02), and a larger probability for steeper slopes. Particularly noticeable is the drop in probability for the very weakest slopes (lower than 0.001).

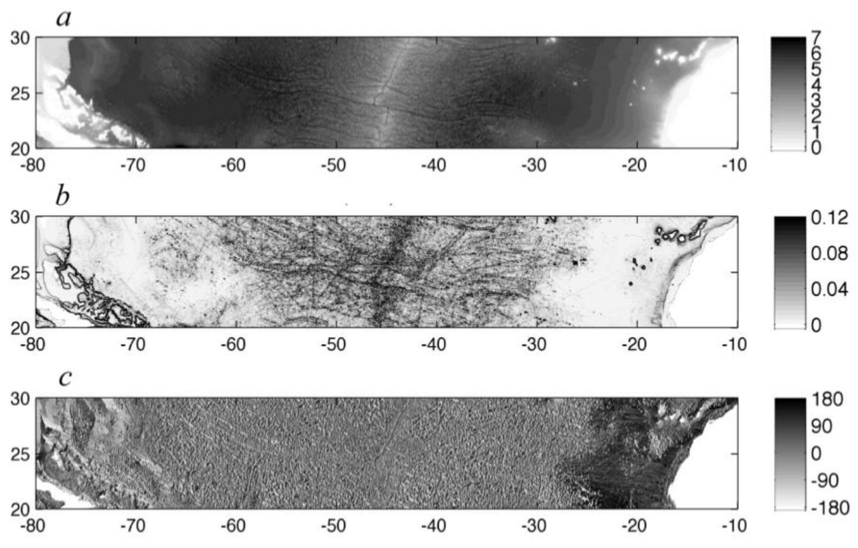

Fig.1. A slice of the Atlantic Ocean, from a later version of the database of [5], at 1/60° resolution.
a – the bathymetry (depth in km); *b* – the steepness γ, derived with formula (1);
c – the horizontal orientation ν, in degrees.

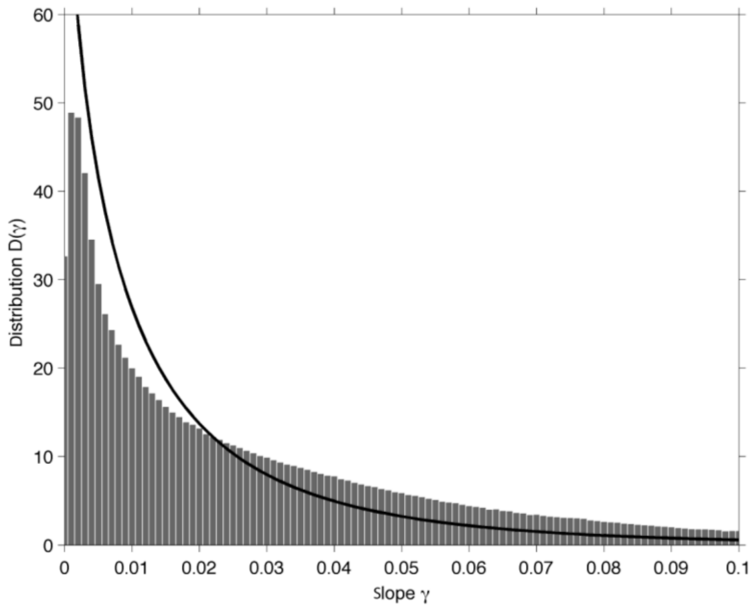

Fig.2. The distribution of steepness γ (grey), based on Fig.1, *b*, taking into account the full three-dimensionality. For comparison, the distribution used by [3], based on sections along lines (black).

We now use this new distribution to calculate the likelihood of critical reflection.

Recapitulation of the criterion of criticality. The expression of criticality was derived by [3]. The situation considered is sketched in Fig.3: an incident ray hits the bathymetry, i.e. its tangential plane, which is oriented arbitrarily. The arrows indicate the direction of energy propagation. The problem is to calculate the direction of energy propagation of the reflected ray. This problem was solved as follows.

Consider an incident internal wave of the form

$$w = \exp i(kx + ly + mz + \omega t)\ ,$$

which upon reflection from a slope $z = \gamma(x\cos\nu + y\sin\nu)$ (x is the west-east coordinate, y is south-north), yields an outgoing wave

$$W = Q\exp i(Kx + Ly + Mz + \omega t)\ .$$

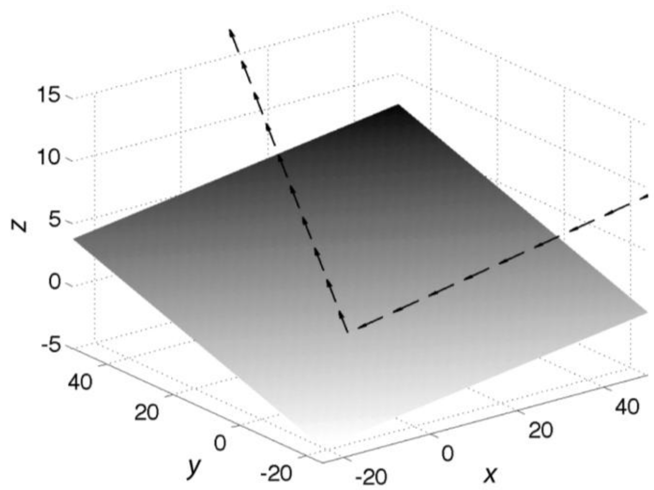

Fig.3. Sketch of an incident ray, impinging on an arbitrarily oriented slope, and the reflected ray.

Here, w and W denote vertical velocities, but the horizontal components can be similarly expressed. The problem is to find Q, K, L, M as functions of k, l, m and γ, ν. The first requirement is that of zero normal velocity at the slope; for this be fulfilled for all x and y, we must have

$$k + m\gamma\cos\nu = K + M\gamma\cos\nu;\ l + m\gamma\sin\nu = L + M\gamma\sin\nu.$$

Furthermore, the incident wave has to satisfy the dispersion relation,

$$Al^2 + 2Blm + Cm^2 + Dk^2 = 0 \qquad (2)$$

with $A = N^2 - \omega^2 + \tilde{f}^2$, $B = f \cdot \tilde{f}$, $C = f^2 - \omega^2$, and $D = N^2 - \omega^2$ [1, 6]. Notice that the so-called non-traditional component (i.e., $\tilde{f}$) is included here. The reflected wave, with (K, L, M), similarly has to satisfy (2).

Writing $M = qm$, the requirements allow us to solve q:

$$q = \frac{A(l + m\gamma\sin\nu)^2 + D(k + m\gamma\cos\nu)^2}{m^2[A(\gamma\sin\nu)^2 + D(\gamma\cos\nu)^2 - 2B\gamma\sin\nu + C]}. \qquad (3)$$

Notice that this expression exhibits an asymmetry in the horizontal plane due to the presence of the non-traditional component $\tilde{f}$. (If we would ignore it, setting $\tilde{f} = 0$, then $A = D$ and $B = 0$ and, as a consequence, the dependence on the horizontal orientation ν disappears.)

In [3] demonstrate that critical reflection occurs if $|q| \to \infty$; thus, the criterion is that the denominator in (3) has to vanish, whence

$$\gamma = \frac{B\sin\nu + [(B\sin\nu)^2 - C(A\sin^2\nu + D\cos^2\nu)]^{1/2}}{A\sin^2\nu + D\cos^2\nu}. \qquad (4)$$

This is the non-traditional criterion for critical reflection. Remarkably, it depends on the horizontal orientation of the plane tangential to the bottom, but not on the horizontal orientation of the incident ray.

Results. As an example we consider the band of diurnal tidal frequencies (O_1, K_1), whose inertial latitudes lie at 27.6 and 30.0°N, respectively.

The most interesting and most relevant case is that of weak stratification, so we consider only the value $N = 2\times10^{-4}$ rad/s (for examples with other values, we refer to [3]. It is the most interesting case because differences between traditional and non-traditional approaches are most conspicuous for weak stratification, and the most relevant one because near the deep-ocean floor, where the reflection takes place, the stratification is weak.

For any given orientation of the slope ν and latitude ϕ, we can translate the range of frequencies (O_1, K_1) into a range of critical slopes $(\gamma_{O_1}, \gamma_{K_1})$, using (4). For that range, we can calculate the probability from the distribution shown in Fig.2. We can repeat this for all angles ν, and take the average. (In principle, one could give a different weight to different angles, following a distribution based on Fig.1, *c*. Such a distribution depends on the orientation of the mid-ocean ridge. For simplicity, we here ignore that dependence and give all angles an equal weight.)

The result is shown in Fig.4 for a range of latitudes. The probability based on the new 3D distribution is shown in black, together with the previous 2D results from [3] in grey. Clearly, the probability of critical reflection is now higher for all latitudes (both under the Traditional Approximation and in a non-traditional approach), except very close to the inertial latitude. The percentual increase compared to the previous 2D approach amounts to 119 % at 24°N, and becomes steadily less towards the inertial latitude, until the increase turns into a decrease.

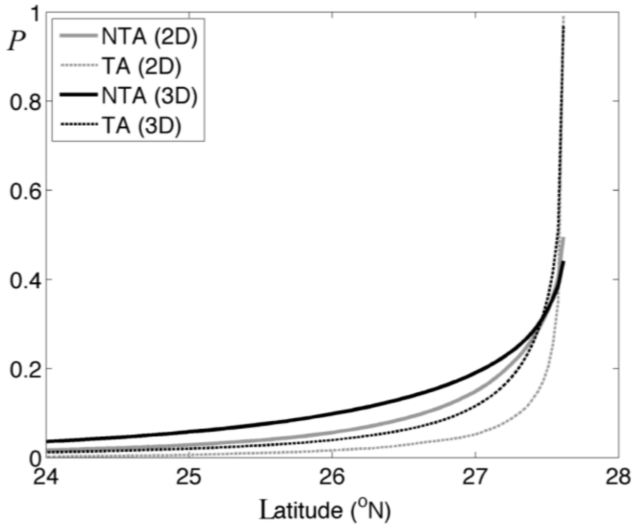

Fig.4. The probability of critical reflection for the range of diurnal tidal frequencies (O_1, K_1), for a range of latitudes up to the inertial latitude of O_1. In grey, the results for the quasi-2D slope distribution adopted by [3]; in black, the new results based on a genuine 3D approach. For both cases, results from the non-traditional approach (NTA) are shown, as well as results with the traditional approximation (TA).

The change with latitude can be explained by the fact that the rays are steeper at lower latitudes, falling in the range where the probability in Fig.2 has increased with respect to the previous 2D approach. Near the inertial latitude, on the other hand, rays tend to be more horizontal and are now less likely to meet a critical slope than in the previous 2D case.

Conclusion. Although the findings obtained earlier by [3] remain qualitatively valid, a significant correction has been uncovered in relation to the calculation of the distribution of bottom slopes in the ocean, as illustrated in Fig.2. We have repeated the calculation for a more northerly area (not shown), with similar results. The upshot is that mild slopes are considerably less likely than a cross-sectional line would suggest. In other words, the fully three-dimensional character of the bathymetry needs to be taken into account.

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