УДК 556.082

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ОПТИМАЛЬНОЕ КОНСТРУИРОВАНИЕ ПРИБОРОВ ДЛЯ ИЗМЕРЕНИЯ ПОКАЗАТЕЛЯ РАССЕЯНИЯ ВОДЫ: ТЕОРЕТИЧЕСКИЕ ОСНОВЫ

Излагаются теоретические основы метода определения полного показателя рассеяния воды (*b*), основанного на измерении характеристик поля излучения широкоугольного источника света. Анализируются три варианта прибора, каждый из которых включает в себя два фотоприемника. Приемники прибора типа 1 измеряют полную облученность (прямым и рассеянным светом) и облученность прямым светом, тогда как приемники прибора типа 2 измеряют полную облученность и облученность рассеянным светом, а приемники прибора типа 3 — облученности прямым и рассеянным светом. Предлагаются оптимальные принципиальные схемы измерителей показателя рассеяния для прибрежной и чистой морских вод. Указан способ нахождения параметров источника и приемников для минимизации погрешностей измерения *b*.

Ключевые слова: вода, свет, показатель рассеяния, измерение, прибор, проектирование.

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OPTIMAL DESIGNING OF INSTRUMENTS FOR DETERMINATION OF THE WATER SCATTERING COEFFICIENT: THE THEORETICAL BACKGROUND

A theoretical background for the methods of determination of the total scattering coefficient (*b*) by measurement of the light field of a wide-angle point source is given. The three versions of the instrument design, each of them including two sensors, are analyzed. The sensors of version 1 measure the total (direct and scattered) and nonscattered irradiances, while the sensors of version 2 measure the total and scattered irradiances, and those of version 3 measure the nonscattered and scattered irradiances. The optimal conceptual versions of the *b*-meter designing method for coastal and pure seawaters are suggested. It is shown how to find the parameters of light source and receiver to ensure the minimal errors of the *b*-measurement.

Key words: water, light, scattering coefficient, measurement, instrument, design.

OCIS Codes: 010.4450 - Oceanic optics

The scattering coefficient *b* (m^{-1}) is one of the main water inherent optical properties (IOPs) which, together with the absorption *a* (m^{-1}) and attenuation *c*, (m^{-1}) coefficients, as well as the volume scattering function (VSF) β(θ) determines propagation of light within natural waters. In addition, the scattering coefficient characterizing water turbidity is regularly measured at water supply enterprises and nuclear power plants. The most widely-spread method of «*in situ*» measurements of the total scattering coefficient *b* is an indirect method based on measuring of either VSF β(θ), or one or several discrete values of VSF [1]. In the first case, *b* is calculated by integration of β(θ) over the entire solid angle ($b = 2\pi\int_0^\pi \beta(\theta)\sin\theta d\theta$), while in the second case it is expressed through β(θ) by an empirical equation, e.g. [2] lg*b* = 0.86 lgβ(6°) – 0.56. The shortcoming of these methods results from the necessity of using rather high-powered light sources and high-sensitive receivers

Dolin L. S., Levin I. M. Optimal designing of instruments for determination of the water scattering coefficient: the theoretical background // Фундаментальная и прикладная гидрофизика. 2016. Т. 9, № 1. С. 83—92.
Dolin L. S., Levin I. M. Optimal designing of instruments for determination of the water scattering coefficient: the theoretical background. *Fundamentalnaya i prikladnaya gidrofizika.* 2016, 9, 1, 83—92.

for measuring weak light signals, as well as the arrangements for angle selection and second processing. During the last two decades field spectrophotometers, AC-9 (WET Labs, Inc.), have been developed and widely used in oceanic, coastal, and lake waters. The attenuation and absorption coefficients are measured simultaneously with AC-9, while the scattering coefficient is not directly measured but estimated from these two parameters as $b = c - a$ [3—5]. Besides, recently the instruments for measurement of the backscattering coefficient $b_b = 2\pi \int_{\pi/2}^{\pi} \beta(\theta)\sin\theta d\theta$ were developed and used in sea expeditions [6, 7]. Analysis of accuracy of the AC-9 measurements [8] shows that the errors of b determinations are typically increasing from 10 to 40 % with the absorption coefficient value a and wavelength. Another method of direct measurement of the forward scattering coefficient was realized using the «Turbido-1» instrument [9] which is a double-channel photometer with a small-size wide-angle source and two receivers, one of which measures irradiance of direct (nonscattered) light, while another one measures the total irradiance at some distance from light source.

The instrument has a base (length of a water layer between protecting glasses of the source and receiver) r_{sr} = 25 cm and allows us to measure b in the range of b = (0.35÷15) m^{-1}, the random measurement error increasing with decrease of b (the maximal standard error for the smallest values of b is about 10 %). For measurement of the scattering coefficient in purer sea waters the instrument based on the same principle as «Turbido-1», should have a base more than 2 m, i.e., such an instrument would be too bulky. Here we perform a comparative analysis of three design versions of a two-channel photometer to determine an optimal b-meter version, which would provide measuring of small values of b with no increase of the base size, and ensure the minimal measurement errors at the given base.

The versions of *b*-meters. Each of the instrument versions consists of the wide-angle light source (S) and two receivers (R_1 and R_2) which include lenses, photo-detectors, amplifications and processing devices. The signals from the photo-detectors after amplification and processing enter the measuring unit in a form of photo-currents I_1 and I_2 (Fig. 1).

In Turbido-1 instrument the photo-currents (I_1, I_2) of the first and second receivers are proportional to the total (E_{tot}) and nonscattered (E_{ns}) irradiances, respectively:

$$I_1 = A_1 E_{tot},\ I_2 = A_2 E_{ns}, \tag{1a, b}$$

where coefficients A_1 and A_2 depend on the optical and electrical parameters of the instrument (areas of photo-detectors and lenses, sensitivity of photocathode etc.), including the amplification factors in the channels of the first and second receivers.

The algorithm of b-determination used by such instrument is grounded on the equations [3]

$$E_{tot} = \frac{P_s \exp(-a r_{sr})}{2\pi(1-\cos\vartheta_s^*) r_{sr}^2},\ E_{ns} = \frac{P_s \exp[-(a+b) r_{sr}]}{2\pi(1-\cos\vartheta_s^*) r_{sr}^2}, \tag{2a, b}$$

which express attenuation of irradiances from a point light source with power P_s and rather large divergence angle $2\vartheta \geq 60°$ in water with the absorption coefficient a and the scattering coefficient b. Eq. (2b) is exact, while Eq. (2a) follows from the solution of the radiative transfer equation (RTE) in small-angle approximation which ignores the effects of the photon multipath dispersion [10]. If $I_2 = I_1$ is set by regulating A_1 and A_2 for $b = 0$ (using twice distillated very pure water, much more transparent than the water with a value of b within the working range of the instrument), it follows from Eqs. (1), (2) that the scattering coefficient can be expressed through the receivers currents:

$$b = -\frac{1}{r_{sr}} \ln \frac{I_2}{I_1}. \tag{3}$$

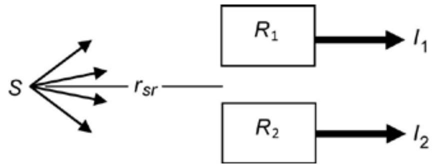

Fig. 1. Simplified basic scheme of the b-meter as a double-channel photometer. S — light source; R_1, R_2 — receivers; r_{sr} — distance between light source and receivers.

«Turbido-1» is a real b–meter, which determines the scattering coefficient b from two signals proportional to the total irradiance E_{tot}, and nonscattered irradiance E_{ns}. We can also imagine the other two types of b-meters, one of which determines b from two signals, proportional to the total E_{tot} and scattered

$$E_s = E_{tot} - E_{ns} \tag{4}$$

irradiances, while another type determines b from two signals proportional to the nonscattered E_{ns} and scattered E_s irradiances. Let us denote the instrument measuring E_{tot} and E_s as «Turbido-2», and the one measuring E_{ns} and E_s as «Turbido-3». For these types of b-meters Eq. (1) takes the form:
for «Turbido-2»

$$I_1 = A_1 E_{tot},\ I_2 = A_2 E_s; \qquad (5a, b)$$

for «Turbido-3»

$$I_1 = A_1 E_{ns},\ I_2 = A_2 E_s. \qquad (6a, b).$$

The relationship between b and the measured irradiances in the «Turbido-2» and «Turbido-3» can be found from the equations:

$$E_s/E_{tot} = 1 - \exp(-br_{sr}),\ E_s/E_{ns} = \exp(br_{sr}) - 1, \qquad (7a, b)$$

which follows from Eqs. (2) and (4). Using Eqs. (5)—(7) and assuming them as applicable for «Turbido-1», $I_2 = I_1$ after regulating A_1 and A_2 for $b = 0$, we obtain the following equation for «Turbido-2»

$$b = -\frac{1}{r_{sr}} \ln\left[1 - \frac{I_2}{I_1}\right], \qquad (8)$$

and for «Turbido-3»

$$b = \frac{1}{r_{sr}} \ln\left[1 + \frac{I_2}{I_1}\right]. \qquad (9)$$

Let us note that to measure the total irradiance E_{tot}, the receiver photo-detector should be placed directly at distance r_{rs} from a wide-angle light source. Measurement of the scattered irradiance E_s can be realized, if a small opaque screen is placed between the light source and photo-detector. This screen will prevent direct light falling from the source on the receiver. To measure the nonscattered irradiance E_{ns}, usually photo-detector is placed behind a small stop situated in the image plane of a lens which is located at distance r_{rs} from the light source. The receiver with the lens can be used also for measurement of the scattered irradiance E_s, if the stop (which is a screen with a small hole) is replaced by the small opaque screen absorbing direct light from the source.

Random measurement errors for different «Turbido» versions. By differentiating Eq. (3) and assuming that the relative error $\delta(I_2/I_1)$ of measuring the ratio (I_2/I_1) equals $2\delta I$ where $I = (I_2 + I_1)/2$, we find that for «Turbido-1» the relative error of b measuring $\delta b = db/b$ can be given by

$$\delta b/\delta I = 2/br_{sr}, \qquad (10)$$

where $\delta I = dI/I$ is an average relative error of the photo-current (I_1, I_2) measurement. It is seen from Eq. (10) that for the given base of device (r_{sr}) the measurement accuracy becomes less with decrease of b.

Differentiating Eqs. (8) and (9) in the same manner as we did with Eq. (3) for «Turbido-1», we find the relative errors of b measurement $\delta b = db/b$ expressed by Eqs. (10*a*) and (10*b*) given in the 4-th line of Table 1 together with Eq. (10) for «Turbido-2» and «Turbido-3».

Table 1 contains the equations for calculation of the scattering coefficient b and the relative random measurement error δb for different instrument versions. Fig. 2 shows in what way the relative error δb depends

Table 1

Equations for calculation of the scattering coefficient (b) and the random measurement error (δb) for different instrument versions

Number of the instrument versions	Turbido-1	Turbido-2	Turbido-3
Measured components of irradiance	$I_1 \sim E_{tot}$ $I_2 \sim E_s$	$I_1 \sim E_{tot}$ $I_2 \sim E_{ns}$	$I_1 \sim E_{ns}$ $I_2 \sim E_s$
Equation for b	$b = -\frac{1}{r_{sr}} \ln\left[\frac{I_2}{I_1}\right]$ (3)	$b = -\frac{1}{r_{sr}} \ln\left[1 - \frac{I_2}{I_1}\right]$ (8)	$b = \frac{1}{r_{sr}} \ln\left[1 + \frac{I_2}{I_1}\right]$ (9)
Equation for δb	$\frac{\delta b}{\delta I} = \frac{2}{br_{sr}}$ (10)	$\frac{\delta b}{\delta I} = \frac{2}{br_{sr}}\left[e^{br_{sr}} - 1\right]$ (10*a*)	$\frac{\delta b}{\delta I} = \frac{2}{br_{sr}}\left[1 - e^{-br_{sr}}\right]$ (10*b*)

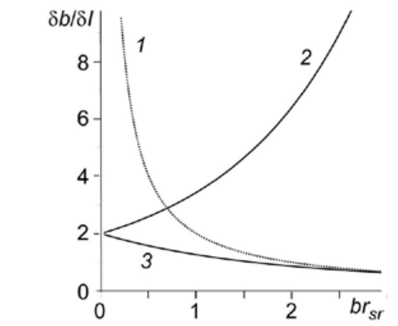

Fig. 2. Ratio of random measurement error of the scattering coefficient (δb) to the photo-current (δI) depending on a dimensionless base of the instrument; the numbers at curves correspond to the instrument versions.

on the «optical base» br_{sr} for the three different versions of the instrument design. One can see that the error $\delta b/\delta I$ decreases with br_{sr} for the instrument version 1 and increases for version 2. For version 3 this error depends on b insignificantly and does not exceed 2 at any values of br_{sr}.

Thus, for the given relative error of the photo-current measurement, the scattering coefficient can be determined with a minimal random error, if one of the photo-detectors measures the nonscattered irradiance, while the other one measures the scattered irradiance (version 3 of the instrument). Version 1 («Turbido-1»), as a e b-meter, is suitable only for turbid waters ($br_{sr} > 1$). The instrument with the photo-detectors measuring the total irradiance and scattered irradiance (version 2), ensures a minimal ratio $\delta b/\delta I$ for pure waters, while in turbid waters its accuracy becomes poor. On the other hand, design of version 2 is simpler for manufacturing as compared with that of version 3. If we intend to measure b, e.g., assuming that the range of $b < 0.3$ m^{-1} and the base $r = 0.3$ m, the random error for the versions 2 and 3 would be almost identical (Fig. 2). Therefore, for such waters preference should be given to the version 2 which is obviously simpler in design.

A systematic measurement error and possibility of its decrease by optimization of the transmitting and receiving directivity patterns. When deriving Eqs. (3), (8), (9), the approximation Eq. (2*a*) has been used, which can lead to systematic errors in determination of the scattering coefficient through these equations. In this section, using the «Turbido-3» system as an example, we evaluate a systematic error of b definition, caused by dependence of E_s/E_{ns} on the water absorption coefficient (a), and on the transmitting ($2\vartheta_s^*$) and receiving ($2\vartheta_r^*$) angles of b-meter. We also analyze the possibility of minimizing the error through optimal choice of the directivity patterns of light source and receiver. To simplify the task, let's assume that the following condition

$$br_{sr} \ll 1, \tag{11}$$

is satisfied, which enables us to use the single scattering approximation for calculation of irradiance at the photo-detector of Receiver R_2. According to Eqs. (6), (7*b*) and (11), the ratio I_2/I_1 in Eq. (9) is expressed as

$$I_2/I_1 = E_s/E_{ns} \approx br_{sr}. \tag{12}$$

To test the accuracy of this expression, we calculate the ratio I_2/I_1 in the single-scattering approximation and compare the obtained equation with Eq. (12).

Fig. 3 illustrates an operating principle of the device for determination of the scattering coefficient through the ratio of currents given by the receivers of scattered and not scattered light.

We believe that the source directivity pattern $D_s(\vartheta_r)$ is axially symmetric and satisfies the normalization condition $D_s(0) = 1$. This pattern is expressed as $D_s(\vartheta_r) = E(r, \vartheta_r)/(E(r, 0)$ through irradiance $E(r, \vartheta_r)$ from the source placed at the surface of a sphere r = const at the point of the polar coordinates r, ϑ_r when $b = 0$.

The electrical currents I_1 and I_2 at the exits of receivers R_1 и R_2 are expressed as

$$I_1 = A_1 E_{ns}(\mathbf{r}_1), \tag{13}$$

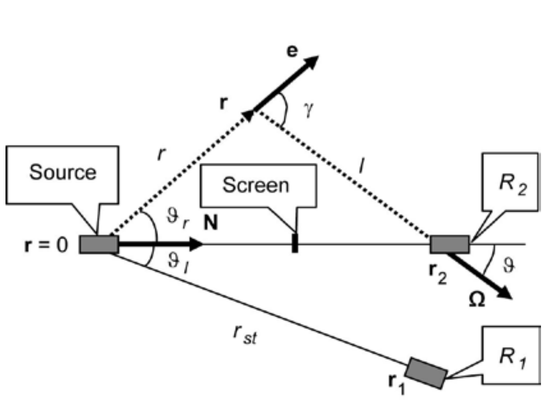

Fig. 3. Schematic diagram illustrating the operating principle of the instrument «Turbido-3»: unit vector **N** characterizes direction of the optical axis of the light source; the receiver of scattered light R_2 is located at point $\mathbf{r}_2 = \mathbf{N}r_{sr}$ on the optical axis of the light source; the receiver of nonscattered light R_1 is located at point $\mathbf{r}_1$ at some angle ϑ_1 to the optical axis of the source; the screen which prevents direct light falling from the source on the receiver, is located between the light source and receiver R_2; the vector $\mathbf{e} = \mathbf{r}/r$ characterizes direction of the direct light entering point $\mathbf{r}$; the unit vector $\boldsymbol{\Omega}$ characterizes the entering direction of the light scattered at point $\mathbf{r}$ to point $\mathbf{r}_2$; ϑ is an angle between the vectors $\boldsymbol{\Omega}$ and $\mathbf{N}$; γ is the scattering angle of the light, entering receiver point $\mathbf{r}_2$ from point $\mathbf{r}$; l is a distance between points $\mathbf{r}$ and $\mathbf{r}_2$.

$$I_2 = A_2 2\pi \int_0^{\pi} L_s(\vartheta) D_r(\vartheta) \sin\vartheta d\vartheta, \qquad (14)$$

where $E_{ns}(\mathbf{r}_1)$ is irradiance from the direct light of source at the receiver R_1 aperture; $L_s(\vartheta)$ is radiance of scattered light at point $\mathbf{r}_2$ in direction Ω; $D_r(\vartheta)$ is the receiver R_2 directivity pattern satisfying the normalization condition $D_r(0) = 1$. To explain the meaning of pattern $D_r(\vartheta)$, note that if the receiver R_2 is used for registration of the light flux from a point light source located at point $\mathbf{r}$ in free space (Fig. 3), a change of angle ϑ would cause changing of the receiver output current $I_2(l, \vartheta)$ in accordance with the equation $I_2(l, \vartheta)/ I_2(l, 0) = D_r(\vartheta)$. Note also, that in case of using a many-element photo-detector, the directivity pattern $D_r(\vartheta)$ would depend not only on the optical arrangement of the instrument, but also on the algorithm of joint processing of the signals from different photo-detector elements. In this case the receiving pattern can take negative values along with the positive ones.

Using Eqs. (13), (14) and solution of RTE in the single scattering approximation for the light field of a point source with the directivity pattern $D_s(\vartheta_r)$ [10], we can express the ratio of the currents I_1 and I_2 as (derivation of Eqs. (15)—(18) see in Appendix A):

$$I_2/I_1 = br_{sr} \cdot AN(\tau), \qquad (15)$$

$$N(\tau) = \frac{1}{2}\int_0^{\pi} x(\gamma) W(\gamma, \tau) \sin\gamma d\gamma, \qquad (16)$$

$$W(\gamma, \tau) = \frac{1}{\sin\gamma}\int_0^{\gamma} D_s(\gamma - \vartheta) D_r(\vartheta) \cdot \exp\varphi(\gamma, \tau, \vartheta) d\vartheta, \qquad (17)$$

$$\varphi(\gamma, \tau, \vartheta) = \tau\left[1 - \cos\vartheta - \frac{\sin\vartheta}{\sin\gamma}(1 - \cos\gamma)\right], \qquad (18)$$

where $A = A_2/[A_1 D_s(\vartheta_1)]$, $\tau = cr_{sr}$, $c = a + b$ is the attenuation coefficient, $x(\gamma)$ is the water scattering phase function (SPF).

The Eq. (15) is reduced to Eq. (12) provided that $AN(\tau) \equiv 1$. The factor A can be reduced to 1 by adjusting the signal gain in the instrument electrical channel. Thus, we can assume that Eqs. (12) and (15) are equivalent when $N = 1$. One can see from Eq. (16) that this condition is true regardless of the form of the scattering phase function only in case

$$W(\gamma, \tau) \equiv 1, \qquad (19)$$

when Eq. (16) meets the normalization condition for the SPF $(1/2)\int_0^{\pi} x(\gamma)\sin\gamma d\gamma = 1$. If the «weight function» W for some values of γ is other than 1, then the condition $N = 1$ is no longer fulfilled, and the scattering coefficient would be expressed with some error by the equation

$$b = r_{sr}^{-1}(I_2 / I_1). \qquad (20)$$

Eq. (17) shows that condition (19) cannot be strictly fulfilled in the finite interval of τ for a fixed form of pattern $D_{s,r}(\vartheta)$. However, for each of the fixed values of τ such patterns can be picked up. In particular, for $\tau \to 0$ the condition $W \to 1$ would be fulfilled for the directivity patterns of the light source and receiver

$$D_s(\vartheta) \equiv 1,\ D_s(\vartheta) = \cos\vartheta, \qquad (21)$$

what can be seen if Eq. (21) is substituted for Eq. (17), and the obtained integral is calculated for $\varphi = 0$.

Thus, for $\tau \to 0$ an optimal meter of the scattering coefficient should have an isotropic light source and a receiver of scattered light with the cosine directivity pattern. It means that a receiver should measure the light flux density in direction **N**, that is the quantity $E_s^+ - E_s^-$, where E_s^+ и E_s^- are irradiances by scattered light of the two sides of a small area (the side turned to the source and the shadow side), placed at some distance r_{sr} and normal to vector **N**.

The simplest receiver with such directivity pattern can be built in a form of the two plane photo-detectors which receive light from the left and right hemispheres, and a device for subtraction of the photo-detectors currents (Fig. 4).

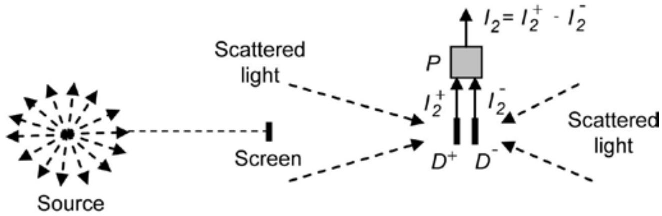

Fig. 4. Schematic diagram illustrating arrangement of the optimal Receiver 2: photo-detector D^+ receives scattered light from the left hemisphere, where the point isotropic light source is located; absorbing screen prevents direct light falling from the source on the receiver D^+; photo-detector D^- receives light from the right hemisphere; currents I_2^+, I_2^- flow to the subtraction device *P*, while current I_2 is taken from the exit of device *P*.

The results of the calculations of function *W* for the instrument with the directivity patterns expressed by Eq. (21) with regard to the dependence of this function on τ are shown in Fig. 5 with solid curves. One can see that the most essential deviations of *W* from 1 fit the scattering angles $\gamma \to \pi$, i.e., the interval of angles where these deviations are large, increases with the attenuation coefficient *c*. This effect is due to the fact that an average path of scattered light between the light source and receiver increases with the scattering angle, while radiance of received light decreases with increase of its path and the attenuation coefficient.

We have also calculated the dependence of *W* on γ for a scattering coefficient meter with the step-patterns of the following type

$$D_s(\vartheta) = \begin{cases} 1, & \vartheta \le \vartheta_s^* \\ 0, & \vartheta > \vartheta_s^* \end{cases}, \quad D_r(\vartheta) = \begin{cases} \cos\vartheta, & \vartheta \le \vartheta_r^* \\ 0, & \vartheta > \vartheta_r^* \end{cases}. \tag{22}$$

Fig. 5 demonstrates an example of this dependence for case $\vartheta_r^* = \pi/2$, when the receiver of scattered light measures irradiance of the area element normal to the receiver optical axis (dotted lines). The calculations show that at $\vartheta_r^* = \pi/2$ and $\vartheta_s^* < \pi/2$, the weight function *W* is still close to 1 in the interval of the scattering angles $0 < \gamma < \vartheta_s^*$, while outside this interval it decreases sharply.

The curves of Fig. 6 give a better understanding about how function *W* depends on the transmitting and receiving angles of *b*-meter.

Comparison of Eqs. (12) and (15) shows that the scattering coefficient can be determined by Eq. (20) with a systematic error $\Delta b = (N - 1)b$. To estimate a random relative error of *b* determination, we use the equation

$$\delta = \Delta b / b = N - 1 \tag{23}$$

and Eq.(16) in which we substitute a model scattering phase function (SPF)

$$x(\gamma) = (1 - 2\tilde{b}_b) 2q\gamma^{-1} \exp(-q\gamma) + 2\tilde{b}_b, \tag{24}$$

where $\tilde{b}_b$ is the backscattering probability, *q* is a parameter of the small-angle part of SPF. These parameters can be expressed through an attenuation coefficient by regressions for the case 2 waters [11]:

$$q = \sqrt{\frac{19.1 \cdot c - 0.95 m^{-1}}{0.34 \cdot c - 0.01 m^{-1}}}, \quad \tilde{b}_b = \frac{0.018 \cdot c}{0.944 \cdot c - 0.048 m^{-1}}. \tag{25}$$

The results of the calculations of a relative error (δ) of *b* — determination by Eqs. (16)—(18), (23)—(25) for the instruments with the patterns of (21) and (22) types are shown in Fig. 7.

It can be seen from Fig. 7 that due to the strongly anisotropic water SPF, a dramatic reduction of *W* for the scattering angles γ > 60 deg does not produce an essential error in determination of the total scattering coefficient.

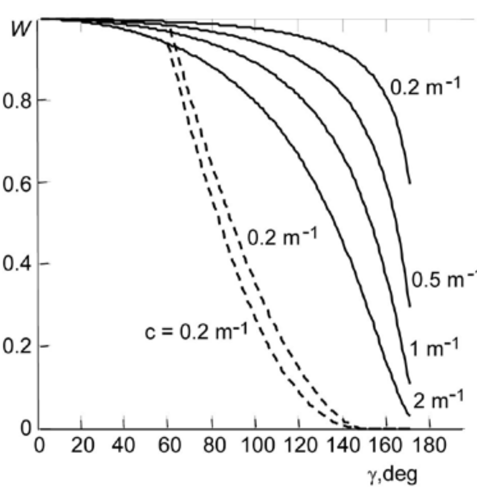

Fig. 5. The weight factor *Wvs* scattering angle γ for an optimal scattering coefficient meter with the directivity patterns of Eq. (21) type (solid lines) and for a similar meter with the directivity patterns of Eq. (22) type and $\vartheta_s^* = \pi/3$, $\vartheta_r^* = \pi/2$ (dotted lines). The distance between the light source and receivers is $r_{sr} = 0.3$ m, the values of the water attenuation coefficient are shown in the figure.

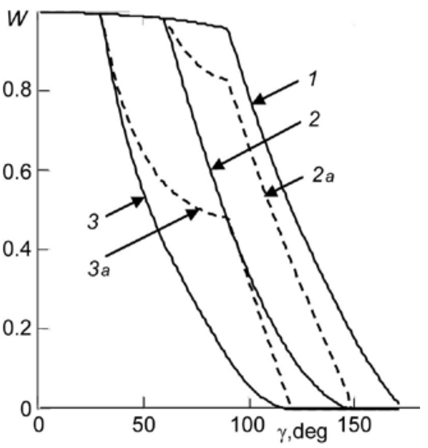

Fig. 6. Dependence of weight factor W on scattering angle γ for b-meter with the directivity patterns of Eq. (22) type ($\tau = 0.15$) and for different values of transmitting and receiving angles: $\vartheta_s^* = \vartheta_r^* = \pi/2$ (*1*); $\vartheta_s^* = \pi/3$, $\vartheta_r^* = \pi/2$ (*2*); $\vartheta_s^* = \pi/2$, $\vartheta_r^* = \pi/3$ (*2a*); $\vartheta_s^* = \pi/6$, $\vartheta_r^* = \pi/2$ (*3*); $\vartheta_s^* = \pi/2$, $\vartheta_r^* = \pi/6$ (*3a*).

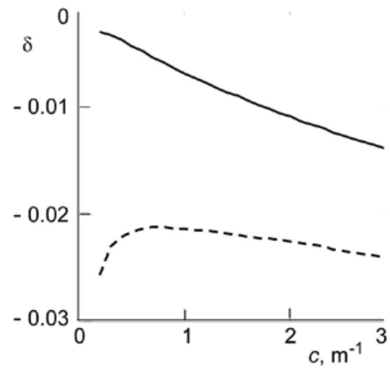

Fig. 7. Relative error of the scattering coefficient determination (δ) vs the attenuation coefficient (c). The solid curve relates to pattern (21), while the dotted one corresponds to pattern (22) with the parameters $\vartheta_s^* = \pi/3$, $\vartheta_r^* = \pi/2$.

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Several design versions of an instrument for measurement of the total water scattering coefficient, as a two-channel photometer with a wide-angle light source are analyzed. The possible ways of the instrument construction improvements targeted at achieving the maximal accuracy of measurement in the given range of water turbidity are suggested. It is shown that the character of dependence between the random measurement error and the instrument base (length of the analyzed water layer) dramatically changes with a type of the light field parameters to be measured. The three versions of the instrument design, each of them including two sensors, are analyzed. The sensors of version 1 («Turbido-1») measure the total (direct and scattered) and nonscattered irradiances, while the sensors of version 2 («Turbido-2») measure the total and scattered irradiances, and the «Turbido-3» version is intended for measurement of nonscattered and scattered irradiances. It is shown that the scattering coefficient can be determined with a minimal random error using version 3 of the instrument. Version 1 is suitable only for turbid waters. Version 2 of the instrument ensures a minimal error for pure waters, while in turbid waters its accuracy becomes poor, still, it is simpler in design as compared to Version 3. In particular, for the waters with the scattering coefficient $b < 0.3$ m^{-1} and the base of the instrument $r = 0.3$ m, the random error for versions 2 and 3 would be almost identical. Therefore, for such waters a preference should be given to Version 2 which is simpler in design. The systematic errors of b-measurement due to the dependence of instrument readings on the water attenuation coefficient and on the transmitting and receiving directivity patterns of b-meter are estimated. It is shown that the systematic error would be minimal when using the instrument «Turbido-3» with the isotropic light source and receiver R_2, which measures the light flux density in the scattered light field, i.e., the difference of scattered irradiances of a small area at both sides (the side turned towards the source, and the shadow side). It was found also that a use of anisotropic light source with rather large transmitting angle and receiver R_2 measuring irradiance of a small area turned to the source does not produce the essential systematic error in determination of the scattering coefficient due to the strongly anisotropic water SPF. (Besides, we give short information about experience of practical realization of the described principles of building a b-meter). Besides, a brief description of practical experience in realization of the above mentioned principles of building a b-meter is presented.

This work was supported by ONR and OSD under SPAWARSYSCEN contract No. N68171-99-M-5352, and the Russian Foundation for Basic Research, projects No. 13-05-00050 and No. 15-45-02610. We thank Gary Gilbert for assistance and support of this work.

Appendix A

Deriving the equations for determination of the water scattering coefficient by ratio of currents which are exits of the receivers of scattered and unscattered light

Let us present the radiative transfer equation (RTE) as

$$(\Omega\nabla + c)L(\mathbf{r},\Omega) = \frac{b}{4\pi}\int_{4\pi} L(\mathbf{r},\Omega)x(\gamma')d\Omega' + Q(\mathbf{r},\Omega), \tag{A1}$$

where $L(\mathbf{r}, \mathbf{\Omega})$ is the radiance of a light field at point $\mathbf{r}$ in direction of the unit vector $\mathbf{\Omega}$, $\gamma' = \arccos(\mathbf{\Omega'\Omega})$ is the angle between vectors $\mathbf{\Omega}'$ and $\mathbf{\Omega}$, $d\Omega'$ is an element of a solid angle, $\Omega\nabla = \Omega_x \frac{\partial}{\partial x} + \Omega_y \frac{\partial}{\partial y} + \Omega_z \frac{\partial}{\partial z}$, $Q(\mathbf{r}, \mathbf{\Omega})$ is a volume density of light sources. As it follows from Eq. (A1), the radiances of nonscattered and single scattered light $L_{ns}(\mathbf{r}, \mathbf{\Omega})$, $L_s(\mathbf{r}, \mathbf{\Omega})$ are expressed through $Q(\mathbf{r}, \mathbf{\Omega})$ by the equations

$$L_{ns} = \int_0^\infty Q\ (\mathbf{r}, \mathbf{\Omega}l, \mathbf{\Omega})\exp(-cl)dl, \tag{A2}$$

$$L_s = \int_0^\infty Q_s\ (\mathbf{r} - \mathbf{\Omega}l, \mathbf{\Omega})\exp(-cl)dl,\ Q_s(\mathbf{r}, \mathbf{\Omega}) = \frac{b}{4\pi}\int_{4\pi} L_{ns}\ (\mathbf{r}, \mathbf{\Omega}')x(\gamma')d\Omega'. \tag{A3}$$

We believe that the light source is a point source with the axial symmetric directivity pattern $D_s(\vartheta)$, and a corresponding function Q is defined as

$$Q(\mathbf{r}, \mathbf{\Omega}) = A_s\delta(\mathbf{r})D_s(\vartheta), \tag{A4}$$

where $\vartheta = \arccos(\mathbf{\Omega N})$ is an angle between the vector $\mathbf{\Omega}$ and the source optical axis $\mathbf{N}$. As it follows from condition $P_s = \iiint_\infty \left[\int_{4\pi} Q(\mathbf{r},\Omega)d\Omega\right]d^3\mathbf{r}$, a factor A_s in the right part of Eq. (A4) is expressed through the source power P_s and its directivity pattern as $A_s = P_s\left[2\pi\int_0^\pi D_s(\vartheta)\sin\vartheta d\vartheta\right]^{-1}$.

In accordance with Eqs. (A2), (A4), the radiance of unscattered light at arbitrary point of space $\mathbf{r}$ is determined by the equations

$$L_{ns}(\mathbf{r}, \mathbf{\Omega}) = E_{ns}(\mathbf{r})\delta(\mathbf{\Omega} - \mathbf{e}),\ E_{ns}(\mathbf{r}) = A_sD_s(\vartheta_r)\exp(-cr)/r^2, \tag{A5}$$

where $E_{ns}(\mathbf{r})$ is irradiance from direct light of the source on the sphere $r = |\mathbf{r}| = \text{const}$ at point $\mathbf{r}$, $\mathbf{e} = \mathbf{r}/r$, the angle $\vartheta_r = \arccos(\mathbf{eN})$ characterizes direction at point $\mathbf{r}$. From Eqs. (A5), (A3) one can find that the volume density of sources of the single scattered light is

$$Q_s(\mathbf{r}, \mathbf{\Omega}) = \frac{b}{4\pi} E_{ns}(\mathbf{r})x(\gamma),\ \gamma = \arccos(\mathbf{\Omega e}), \tag{A6}$$

while radiance of the single scattered light at the receiving point $\mathbf{r}_2$ is defined as

$$L_s(\vartheta) = \frac{b}{4\pi}\int_0^\infty E_{ns}\ (\mathbf{r}_2 - \mathbf{\Omega}l)x(\gamma)\exp(-cl)dl = \frac{bA_s}{4\pi}\int_0^\infty D_s(\vartheta_r)x(\gamma)\exp\left[-c(r+l)\right]\frac{dl}{r^2}, \tag{A7}$$

in which the values of ϑ_r, γ, r (see Fig. 3) are expressed through r_{sr}, ϑ, l by equations

$$r_{sr}/\sin\gamma = r/\sin\vartheta = l/\sin\vartheta_r,\ \gamma = \vartheta_r + \vartheta. \tag{A8}$$

Replacing the integration variable by in Eq. (A7) and using Eqs. (A8), we get:

$$L_s(\vartheta) = \frac{bA_s}{4\pi r_{sr}\sin\vartheta}\int_{\vartheta_r}^{\pi} D_s(\gamma - \vartheta)x(\gamma)\exp\left[-cr_{sr}f(\gamma,\vartheta)\right]d\gamma, \tag{A9}$$

$$f(\gamma,\vartheta) = \frac{1}{\sin\gamma}\left[\sin(\gamma - \vartheta) + \sin\vartheta\right]. \tag{A10}$$

From Eqs. (13), (A5) it follows:

$$I_1 = A_1A_sD_s(\vartheta_1)r_{sr}^{-2}\exp(-cr_{sr}), \tag{A11}$$

while to determine the current I_2, Eq. (A9) should be substituted for Eq. (14). Then we obtain Eqs. (15)—(18) for the ratio I_2/I_1.

Appendix B

The construction of «Turbido-1» and «Turbido-2»

To better understand how to design different types of *b*-meter, we will describe construction of the two already existing *b*-meters: «Turbido-1» and «Turbido-2».

The instrument «Turbido-1» was intended for quality control of water treatment. Around 100 test units of «Turbido-1» were manufactured and used at water supply enterprises and nuclear power plants. The unit of «Turbido-1» consists of two hermetic boxes, transmitter box and receiver one, connected by a rigid framework which does not prevent water from flowing into the space between the boxes; however, it protects the receivers from the outer lighting. In the transmitter box a light source (light diode) is placed directly in front of the protecting glass. The radiant intensity is constant in a range of angles not exceeding 60°. In the receiver box a lens with a hole is placed directly behind the second protecting glass. In this box there are also two coaxial identical photo-detectors. The first of them, placed inside the lens hole, measures the total (nonscattered and scattered) irradiance E_{tot}, while the second one is located behind a small circular stop situated in the image plane of the lens. This photo-detector measures only the nonscattered irradiance E_{ns}. The spot size (0.25 mm) is almost equal to that of the light source image, therefore the entire nonscattered light passing through the lens, falls on the photo-detector. The instrument base $\boldsymbol{r}_{\mathbf{sr}}$ = 25 cm. The limits of measurements in the spectral region around 580 nm are $b = (0.35 \div 15)$ m^{-1}. The measurement error is about 3—10 %, decreasing with *b*. An external view of «Turbido-1» is shown in Fig. B1.

Since the source directivity diagram of the instrument is not wide enough ($2\vartheta_s^* = 60$ deg), then, strictly speaking, for this meter Eq. 2*a* should be replaced by

$$E_{tot} = \frac{P_s \exp(-(a + b_b) r_{sr})}{2\pi(1 - \cos\vartheta_s^*) r_{sr}^2}, \qquad \text{(B1)}$$

where $\tilde{b}_b$ is the backscattering probability, and Eq. 3 should be replaced by

$$b = -[r_{sr}(1 - \tilde{b}_b)]^{-1}[\ln(I_2 / I_1)]. \qquad \text{(B2)}$$

The detailed analysis carried out [9] on the basis of the long-term measurements of inherent optical properties in different regions of the World Ocean [12] shows that the maximal additional error of *b*-determination by Eq. (B2) for $b > 0.05$ m^{-1} is about 4 %, if we set $(1 - \tilde{b}_b)^{-1} = 1.06$. Another systematic error with the «Turbido-1» in principle can occur due to violation of the Bouguer law (Eq. 2*b*), as a result of the scattered light falling on the spot. The analysis given in [9] shows that this error is absent in any water if the angle $\varphi = d/r_{sr} < 0.015$ rad (*d* is diameter of the light source). With «Turbido-1» $\varphi = 0.01$.

The single laboratory model of Version 2 of the instrument for measurement of the water scattering coefficient *b* with the directivity patterns of source and receiver

$$D_s(\vartheta_s < 120°) = 1,\ D_s(\vartheta_s > 120°) = 0,\ D_r(\vartheta_r < 72°) = \cos\vartheta_r,\ D_r(\vartheta_r > 72°) = 0 \qquad \text{(B3)}$$

and the base r_{sr} = 32 cm («Turbido-2») was designed and manufactured at the Institute of Applied Physics, Russian Academy of Sciences.

A general view of the device is shown in Fig. B2. The picture of the b-meter was taken in such a way that its lower part is seen in the foreground while the upper part is in the background. The case of the device is made

Fig. B1. External view of «Turbido-1».

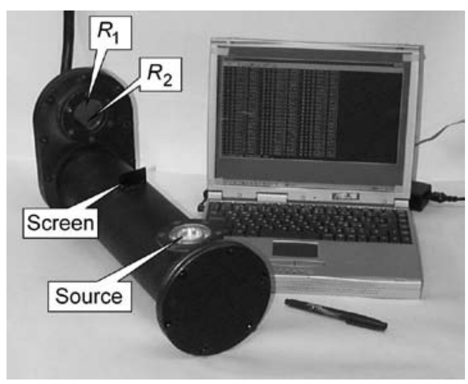

Fig. B2. External view of «Turbido-2».

as a cylinder with two flanges and covers at both ends. The cable enters the case through the upper cover. There is a glass window of the light source in the lateral case wall near the lower flange. The source has three groups of light diodes differing in radiation wavelength (eight pieces in a group). The receiving glass window is on the lateral bugle of the upper flange. Photo-detector receivers are in the cavity between the flange and upper cover at different distances from the lateral case wall. A screen which protects receiver R_2 from the source direct light is attached to the lateral wall of the central part of the case. Therefore, only scattered light is incident upon receiver R_2, while receiver R_1 takes both scattered and nonscattered light.

The instrument measures the total scattering coefficient of seawater at three wavelengths (475, 525 and 590 nm) in the range of $b = 0.05 \div 3$ m^{-1} at depths up to 100 m.

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Статья поступила в редакцию 02.11.2015 г.