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О МЕТОДЕ ЗЕРКАЛЬНЫХ ТОЧЕК

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Для расчёта характеристик света, отражённого от взволнованной морской поверхности, применяются два подхода. Первый есть так называемый «Метод стохастических распределённых площадок», который позволяет рассчитывать только средние характеристики отражённого света. Второй — так называемый «Метод зеркальных точек», который для средних характеристик даёт тот же самый результат, что и метод стохастических распределённых площадок, однако метод зеркальных точек может быть применён для расчёта флуктуаций характеристик отражённого света. Кроме того, метод зеркальных точек имеет дело такими визуально воспринимаемыми геометрическими параметрами морской поверхности, какими являются расположение и число зеркальных точек, кривизны в зеркальных точках, которые пропорциональны размерам бликов. Эти параметры могут быть определены из снимки бликов и могут служить для определения состояния волнения посредством решения прямой и обратной задачи, которая сформулирована в виде интегрального уравнения Фредгольма 1-го рода. Из этих параметров может быть определён также мгновенный рельеф морской поверхности, что открывает путь к постановке и решению новых задач. Корректность полученных теоретических выражений проверяется численным моделированием.

Ключевые слова: солнечные блики, зеркальные точки, полная кривизна, морская поверхность, обратные задачи.

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ABOUT THE METHOD OF SPECULAR POINTS

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For calculation of the characteristics of light, reflected from a wavy sea surface two approach is commonly used. The first, so called “Method of statistically distributed facets”, that allows only calculating the average values of reflected light characteristics. The second: “Method of specular points” that gives the same result for average of characteristics, but it enables to calculate the fluctuations of theirs. Moreover, the method specular points deals with a visually sensing parameters of surface such as number, location and curvatures of specular points. These parameters that can be derived from glint images allow determining sea surface wave state by solving of direct and inverse problem. Besides, from this parameters can be determined an instantaneous relief of the sea surface, that opens opportunity for formulation and solution new problems.

Key words: Sun glints, specular points, total curvature, sea surface, inverse problem.

Introduction. For calculation of the statistical characteristics of light, reflected from a wavy sea surface $z = \zeta(x, y)$ two approach is commonly used. The first, so called “Method of statistically distributed facets” (MSDF), that allows only calculating the average values of reflected light characteristics [1]. Let as, sea surface is illuminated by parallel light beams incident in the direction of unite vector \vec{s}_0 and observation of the surface is made in the direction of unite vector \vec{s} (fig. 1). Then MSDF states that the angular distribution of the reflection coefficient $\langle r \rangle$ is governed by the distribution $W_2(\gamma_x, \gamma_y)$ of surface slopes $\vec{\gamma} = (\gamma_x, \gamma_y) = \nabla \zeta(x, y)$ and is determined by expression:

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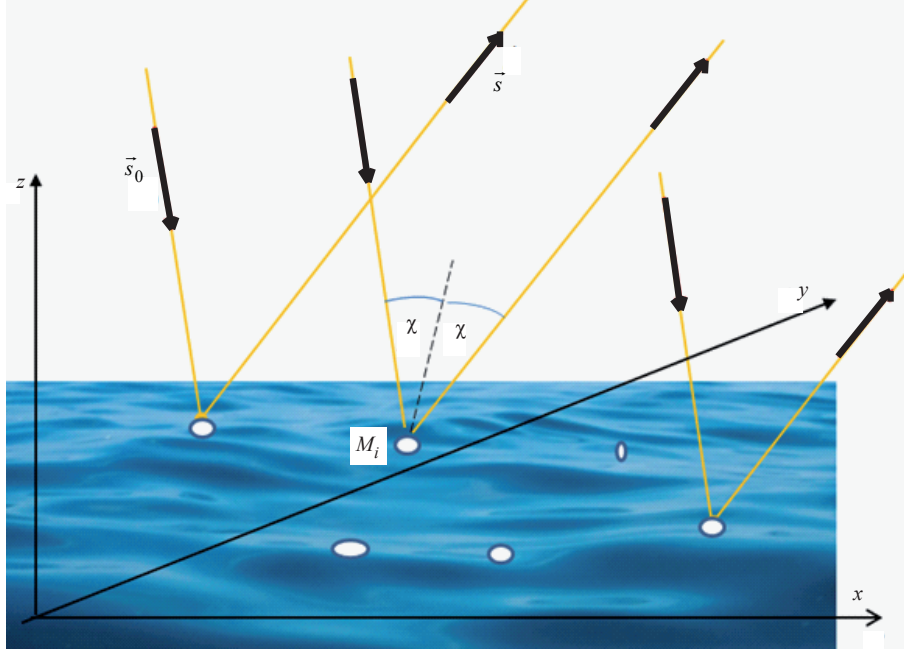


Fig. 1. Geometry of reflection and observation of parallel light beams.

Рис. 1. Геометрия отражения и наблюдения параллельного пучка световых лучей.

$$\langle r \rangle = \frac{\pi V(\chi)}{4 |s_{0z}| s_z} \left(\frac{q}{q_z} \right)^4 W_2(\gamma_x, \gamma_y). \quad (1)$$

Here, $\vec{q} = \vec{s} - \vec{s}_0$, $q_z = s_z - s_{0z}$, $q = |\vec{q}|$; $V(\chi)$ — Fresnel reflection coefficient at the local angle of incidence χ determining by directions \vec{s}_0 and \vec{s} . Note, that (1) is correct if the directions of \vec{s}_0 and \vec{s} close to vertical and the surface slopes $|\gamma_x|$, $|\gamma_y|$ are small. In general case the right side of (1) must be multiplied by $Q(\vec{s}_0, \vec{s})$ — shadowing function which indicates the portion of surface that simultaneously illuminated and observed [2].

The second approach is “Method of specular points” (MSP) that determines the radiance of surface as a sum of intensities from every specular points (SP) inside of considered part of surface. According to MSP a random realization of reflection coefficient r of surface is determined by formulae:

$$r = r(\vec{s}_0, \vec{s}) = \frac{K}{S_\perp} \sum_{i=1}^{N_s} \frac{1}{|\Omega_i|} = \frac{K}{S_\perp \sqrt{3H}} \sum_{i=1}^{N_s} X_i, \quad (2)$$

where, $K = \frac{\pi V(\chi)}{|\Omega_i|}$ depends on \vec{s}_0 , \vec{s} and Fresnel reflection coefficient $V(\chi) = V(\vec{s}_0, \vec{s})$ at the local angle of incidence χ ; S_\perp — the area of projection of observed part of surface S ; $X_i = \frac{\sqrt{3H}}{|\Omega_i|}$ — “dimensionless radius of curvature” — the quantity which is reciprocal to the total curvature Ω_i at the SP, $M_i = (x_i, y_i, \zeta(x_i, y_i))$, with gradient $\vec{\gamma} = (\gamma_x, \gamma_y)$:

$$\Omega_i = \frac{\omega_i}{g} \quad \left(\omega_i = \zeta_{xx}(x_i, y_i) \zeta_{yy}(x_i, y_i) - \zeta_{xy}^2(x_i, y_i), g = (1 + \gamma_x^2 + \gamma_y^2)^2 \right).$$

The parameter H is defined as, $3H = \langle \omega \rangle$, where the averaging is produced on all points of the surface $z = \zeta(x, y)$ [2]. The SP, are defined from the roots of the system of equations:

$$\begin{cases} \zeta_x(x, y) = \gamma_x \\ \zeta_y(x, y) = \gamma_y \end{cases},$$

where, $\gamma_x = \frac{s_x - s_{0x}}{s_z - s_{0z}}$, $\gamma_y = \frac{s_y - s_{0y}}{s_z - s_{0z}} = \left(\frac{q}{q_z} \right)$, $\left(\frac{q}{q_z} = \sqrt{1 + \gamma_x^2 + \gamma_y^2} \right)$.

Note that the formula (2) was obtained on the assumption that the principal radii of curvature r_1 and r_2 at the specular point are much smaller than the distance from the specular point to the point source and point receiver. Moreover, it is impossible to get a flat surface as a limit from the formula (2). To make such a transition it is necessary to use the general reflective formulas [2], where the source and the receiver must be taken on the finite distance and further to approach $r_1 \rightarrow \infty$ or $r_1 \rightarrow \infty$ and $r_2 \rightarrow \infty$. Then as a result we will get, that the divergence of a falling beam does not change in reflection from a flat surface.

In the sum (2) the random variables are the radii of curvatures $X_1, X_2, X_3, \dots, X_{N_s}$ which are changed from the realization to realization of the surface, and the number of SP, N_s , that fall into the observed section of surface S . Note, that the expression (2) sums up the intensities of the light waves, which come from the different SP, therefore it is correct either for the incoherent light or when the wavelength of the light λ is considerably less than the root-mean-square height $\sigma = \sqrt{\langle \zeta^2(x, y) \rangle}$ of the surface roughness, i.e., when $\lambda \ll \sigma$.

Averaging the (2) we obtain:

$$\langle r \rangle = \frac{K}{S_{\perp}} \langle N_s \rangle \left\langle \frac{1}{|\Omega|} \right\rangle = K \langle N_1 \rangle \left\langle \frac{1}{|\Omega|} \right\rangle,$$

where $\langle N_1 \rangle = \frac{\langle N_s \rangle}{S_{\perp}}$ is the density of SP (average number of SP per unit area), $\left\langle \frac{1}{|\Omega|} \right\rangle$ is the average of $\frac{1}{|\Omega_i|}$.

Using the results from [3, 4] in [5] it is proved that:

$$\left(\frac{q}{q_z} \right)^4 W_2(\gamma_x, \gamma_y) = \langle N_1 \rangle \left\langle \frac{1}{|\Omega|} \right\rangle. \quad (3)$$

The equality (3) is pure geometric-statistical relationship.

Equality (3) indicates that the MSDF and MSP give the same result for average radiance of surface. While MSDF allows calculation only of average radiance, MSP can be used to calculate the fluctuation of radiance. Consequently, in approach of MSP the study of statistical characteristics of the reflected light is reduced to the statistical analysis of geometrical characteristics of the surface, such as, the number of SP, N_s , and the “radii of curvature” $\rho_i = 1/|\Omega_i|$ at the SP. An advantage of the MSP is that it deals with intuitively sensed characteristics of the surface — the SP.

Distribution density of the total curvature at the SP. For the uniform Gaussian surface $z = \zeta(x, y)$ the distribution density of the total curvature Ω at SP, with the gradients $\gamma_x = \zeta_x(x, y) = 0$, $\gamma_y = \zeta_y(x, y) = 0$ was obtained by Longuet—Higgins [4]. In this work the distribution density $W(\omega)$ of the curvature $\omega = \Omega \Big|_{\substack{\gamma_x=0 \\ \gamma_y=0}} = \zeta_{xx}(x, y)\gamma_{yy}(x, y) - \gamma_{xy}^2(x, y)$ at the SP with zero slopes was expressed by contour integral.

Comparatively simple expression for the density $W(\omega)$ in the form of integral representation, including the error function and which is convenient for the practical calculations, was derived by Gardashov [6]. Correctness of the expression of $W(\omega)$ was tested by numerical modelling [7] and processing photographs of Sun glints [8].

Being written for the dimensionless curvature, $\bar{\omega} = \omega/3H$, it takes the form:

$$W_{-}(\bar{\omega}) = \frac{1}{\Phi(t)} \frac{(t^2 - t + 1)^{5/4}}{\sqrt{t(t-1)}} (-\bar{\omega}) \exp\left(-\bar{\omega}\sqrt{t^2 - t + 1}\right) \int_0^{\pi/2} \frac{\exp(\bar{\omega}m(\alpha))}{\sqrt{m(\alpha)}} d\alpha$$

for $\bar{\omega} < 0$,

$$W_{+}(\bar{\omega}) = \frac{1}{\Phi(t)} \frac{(t^2 - t + 1)^{5/4}}{\sqrt{t(t-1)}} (-\bar{\omega}) \exp\left(-\bar{\omega}\sqrt{t^2 - t + 1}\right) \times \int_0^{\pi/2} \frac{\exp(\bar{\omega}m(\alpha))}{\sqrt{m(\alpha)}} \left[1 - F\left(\sqrt{m(\alpha)}\right)\right] d\alpha \quad (4)$$

for $\bar{\omega} < 0$.

Here, $F(t) = \frac{2}{\sqrt{\pi}} \int_0^t \exp(-\tau^2) d\tau$ is error function and $m(\alpha) = \frac{(t+1)\sqrt{t^2 - t + 1}}{t} (1 - k \sin^2 \alpha)$, $k = \sqrt{\frac{1-2t}{1-t^2}}$.

Function $\Phi(t)$ is expressed by elliptical integrals

$$K(k) = \int_0^{\pi/2} \frac{1}{\sqrt{1-k^2 \sin^2 \alpha}} d\alpha \text{ and } E(k) = \int_0^{\pi/2} \sqrt{1-k^2 \sin^2 \alpha} d\alpha, \text{ as}$$

$$\Phi(t) = \sqrt{1-t^2} E(k) - \sqrt{\frac{1-t}{1+t}} t K(k).$$

Actually, $\Phi(t)$ is a very slowly monotonically decreasing function at the interval $[0, 1/2]$ with the maximum and minimum values $\Phi(0) = 1$ and $\left(\frac{1}{2}\right) = \frac{\pi}{2\sqrt{3}} \approx 0.907$, respectively [4]. The parameter t is determined through the moments of the energy spectrum of surface (waves) and may vary at the interval $[0, 1/2]$. The value $t=0$ corresponds to the case, when waves are formed by two systems of waves, which are intersected at a small angle. The value $t=1/2$ can be realized in different circumstances; for example, when waves is isotropic, or when angular energy distribution in the wave spectrum has a sharp peak.

Note that the asymptotic expression of $W(\bar{\omega})$ and its expansion around $\bar{\omega} = 0$ were derived in [7] and [8].

Since the distribution $W(\bar{\omega})$ is already found, the distribution of the dimensionless curvature $\bar{\Omega} = \frac{\Omega}{\sqrt{3H}} = \frac{\bar{\omega}}{g}$ and the $X = \frac{1}{|\bar{\Omega}|}$ at SP, with the gradients $\gamma_x = \zeta_x(x, y)$, $\gamma_y = \zeta_y(x, y)$ can be determined simply from the relationships:

$$W_{\Omega}(\bar{\Omega}) = g W(g\bar{\Omega}),$$

$$W_X(X) = \frac{g}{X^2} \left[W_- \left(-\frac{g}{X} \right) + W_+ \left(\frac{g}{X} \right) \right]. \quad (5)$$

Then, for the average value of X we have:

$$\langle X \rangle = \int_0^{+\infty} X W_X(X) dX = \frac{\pi g \sqrt{t^2 - t + 1}}{2\Phi(t)}$$

At $g=1$ for $t=0$, $t=1/4$, and $t=1/2$, correspondingly, we find:

$$\langle X \rangle = \frac{\pi}{2\Phi(0)} = \frac{\pi}{2} \approx 1.57, \quad \langle X \rangle = \frac{\pi}{2\Phi\left(\frac{1}{4}\right)} = \frac{\sqrt{13}}{4} \approx 1.5277 \text{ and } \langle X \rangle = \frac{\pi}{2\Phi\left(\frac{1}{2}\right)} \frac{\sqrt{3}}{2} \approx 1.5.$$

The second moment $\langle X^2 \rangle$, consequently, the dispersion $\sigma_X^2 = \langle X^2 \rangle - \langle X \rangle^2 = +\infty$.

As it follows from the results of the work [4], the SP density is determined by the formula:

$$\langle N_1 \rangle = \frac{2\sqrt{3H}}{\pi\sqrt{t^2 - t + 1}} \Phi(t) W_2(\gamma_x, \gamma_y).$$

The graph of the distribution density $W(\bar{\omega})$ is represented in fig. 2.

Distribution density of the reflected light radiance. Distribution density $W_r(r)$ of the radiance r of the surface, can be determined through the characteristic function $\beta(u)$ of the random variable X and the distribution density $W_N(N_s)$ of the number of the SP, N_s , that fall into the observed section of surface. For the characteristic function we have:

$$\beta(u) = \int_0^{+\infty} e^{iuX} W_X(X) dX.$$

Then, if radii of curvature at the SP, $X_1, X_2, X_3, \dots, X_{N_s}$ and N_s are independent among themselves, the distribution density of the sum

$$Z = \sum_{i=1}^{N_s} X_i \quad (6)$$

can be expressed through the module $|\beta(u)| = \sqrt{\beta_1^2(u) + \beta_2^2(u)}$ and the argument $\varphi(u) = \arg \beta(u) = \text{Arctan} \frac{\beta_2(u)}{\beta_1(u)}$ of the characteristic function $\beta(u) = \beta_1(u) + i\beta_2(u)$ according to the

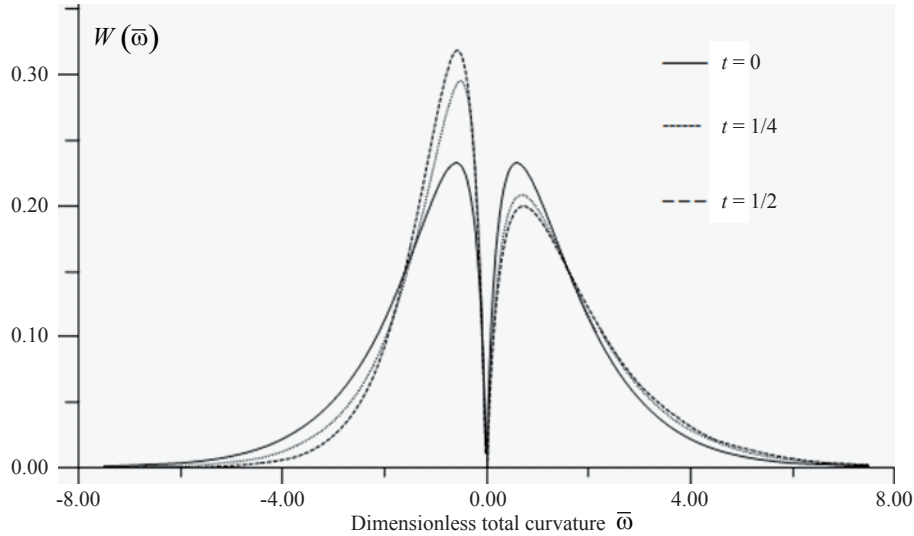


Fig. 2. The distribution density $W(\bar{\omega})$ of dimensionless total curvature $\bar{\omega}$ at the specular points of the Gaussian uniform surface, calculated by the formula (4).

Рис. 2. Плотность распределения $W(\bar{\omega})$ безразмерной полной кривизны $\bar{\omega}$ в зеркальных точках гауссовой однородной поверхности, вычисленной по формуле (4).

formula:

$$W_Z(Z) = \int_0^{+\infty} G(Z, N_S) W_N(N_S) dN_S, \quad (7)$$

where,

$$G(Z, N) = \frac{1}{\pi} \int_0^{+\infty} |\beta(u)|^N \cos(N\phi(u) - uZ) du. \quad (8)$$

In particular case, when the number of SP does not change from realizations to realization and is equal to the number, N i.e., $W_N(N_S) = \delta(N_S - N)$, from (7) we obtain:

$$W_Z(Z) = G(Z, N),$$

which specifies a mean of function $G(Z, N)$, as the distribution of Z when the number of SP, N_S , in the sum (6) is fixed and equal to N . In addition, if $N_S = N = 1$ then $Z = X$ and $G(X, 1) = W_X(X)$.

The curve of the distribution function $W_X(X)$ calculated by the formula (5) at $t = 1/4$ and obtained from numerical modelling and processing Sun glint photo (fig. 4) are given in fig. 3.

As we see from fig. 3, there is good closeness between theoretical and experimental distributions. We think that the main causes of deviations are: 1) the difference between the actual waved water surface and Gaussian surface; 2) the distinction of the Sun from a point light source. Indeed, in closely inspecting the fig. 3, we see that some large glints are formed through confluence of small glints. It tells us, that if we produce experiment with light source which has a solid angle essentially smaller than the Sun, then some large glints will be resolved. Thereat, the number of lager glints will decrease and the number of small glints will increase. As the result, the histogram in fig. 3 (dashed curve) will come closer to theoretical curve. Note that the camera that was used here has a high spatial-temporal resolution therefore the size of the resolution spot is much smaller than the size of the sunglint.

The behavior of the characteristic function $\beta(u)$ is shown in fig. 5.

The curves of the function $G(Z, N)$ for the values of variable $N = 1, 2, 3$ at $t = 1/4$ are given in fig. 6.

Formulation and solution of the invers problem. Relationship (7) can be considered as an integral equation for determining the unknown distribution density, $W_N(N_S)$, of the SP on the given density $W_Z(Z)$. As it was noted in [9], if the average $\left\langle \frac{1}{|\Omega|} \right\rangle$ (or the average of number of SP, $\langle N_S \rangle$) is determined in any manner,

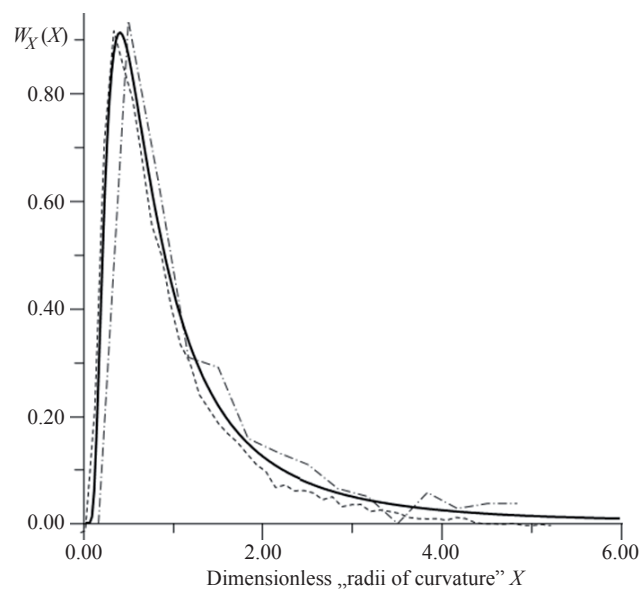


Fig. 3. Theoretical distribution density $W_X(X)$ (solid curve), calculated on (5) and its curves, obtained by the numerical modelling (dashed curve) and by processing the images of the Sun glitters (dotted curve).

Рис. 3. Теоретическая плотность распределения $W_X(X)$ (сплошная), вычисленной по (5), и её кривых, полученных в результате численного моделирования (пунктирная) и в результате обработки изображений солнечных бликов (точечная).



Fig. 4. Sun glints images on the waved basin water surface.

Рис. 4. Изображение солнечных бликов на взволнованной поверхности бассейна.

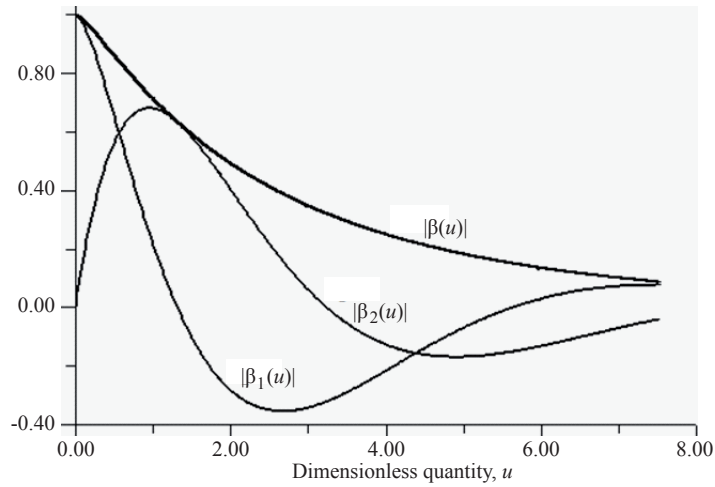


Fig. 5. Real $\beta_1(u)$ and imaginary $\beta_2(u)$ parts and module $|\beta(u)|$ of characteristic function $\beta(u)$.

Рис. 5. Реальная $\beta_1(u)$, мнимая $\beta_2(u)$ части и модуль $|\beta(u)|$ характеристической функции $\beta(u)$.

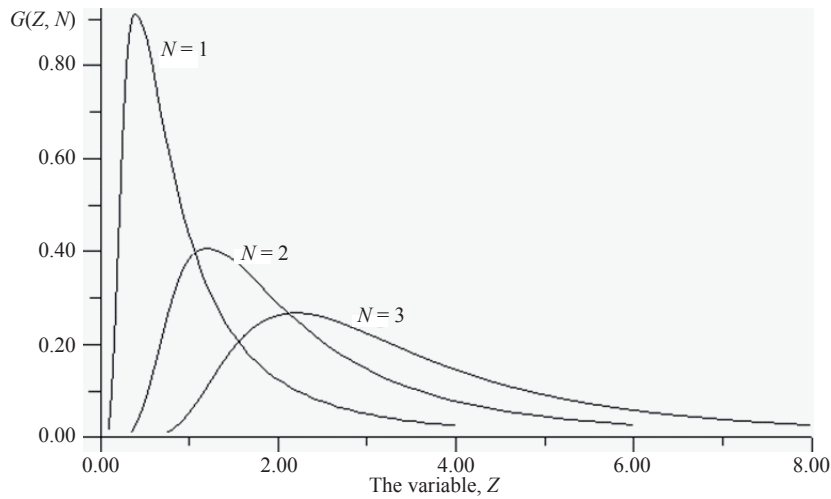


Fig. 6. The function $G(Z, N)$ for values $N = 1, 2, 3$ at $t = 1/4$.

Рис. 6. Функция $G(Z, N)$ для значений $N = 1, 2, 3$ в точке $t = 1/4$.

i.e., is apriori known, then the distribution $W_Z(Z)$ of the quantity $Z = \frac{S_{\perp} \sqrt{3H}}{K} r$ can be obtained simply according to the formula:

$$W_Z(Z) = W_r\left(\frac{K}{S_{\perp} \sqrt{3H}}\right) \frac{K}{S_{\perp} \sqrt{3H}}$$

since, in this case K is a known quantity.

Thus, the equation (7) formulates the inverse problem in the form of Fredholm integral equation of the 1st kind for determining the distribution density $W_N(N_s)$ of the number of the SP, N_s , on the known distribution density $W_r(r)$, of the brightness of reflected light r .

In reality, equation (7) has an algebraic structure. The selection of integral form (7) is connected with the following circumstance: although the number of the SP, N is an integer, however, formally the kernel of equation (7), as can be seen from (8), is determined for all the real N . This fact allows us to take the number of sampling for N a different one from the number of SP and equal to the number of sampling on Z (respectively, on r) in inversion of equation (7).

It is well known, that the equation (7) forms an *Ill-Posed Problems* problem and is solved by the special methods of regularization [10].

Note, that in practical application of the stated above method for determining the distribution density, $W_N(N_s)$, from the measured density, $W_r(r)$, the difficulty appears if both averages: $\left\langle \frac{1}{|\Omega|} \right\rangle$ and $\langle N_s \rangle$ are a priori unknown. However, this difficulty can be overcome by using the fact that for the different N , the curves $G(Z, N)$ noticeably differ in the form, i.e., on the asymmetry and the excess. For the determination $\langle N_s \rangle$, for example, it is possible to use the following criterion: $\langle N_s \rangle$ can be defined as the value $N = \langle N_s \rangle$, for which the measured and theoretically calculated densities $W_r(r)$ differ minimally.

As it follows from (2) and (8)

$$r = \frac{K}{S_{\perp} \sqrt{3H}} Z, \quad \langle r \rangle = \frac{K}{S_{\perp} \sqrt{3H}} \langle Z \rangle, \quad \langle Z \rangle = \langle X \rangle \langle N_s \rangle.$$

Consequently, we obtain the following relationship between the dimensionless brightness $\bar{r} = \frac{r}{\langle r \rangle}$ and the quantity Z :

$$\bar{r} = \frac{1}{\langle X \rangle \langle N_s \rangle} Z,$$

where, for the certainty in the first approximation, it is possible to take the intermediate value of $\langle X \rangle = 1.5277$.

Some results of solution of direct and inverse problem on the procedure developed in [11, 12] are presented in fig. 7—10.

Thus, checking the inverse method both by numerical and *in-situ* experiments shows its fitness for work. Moreover, due to use of fractional values of the number of SP the accuracy of the method can be markedly improved.

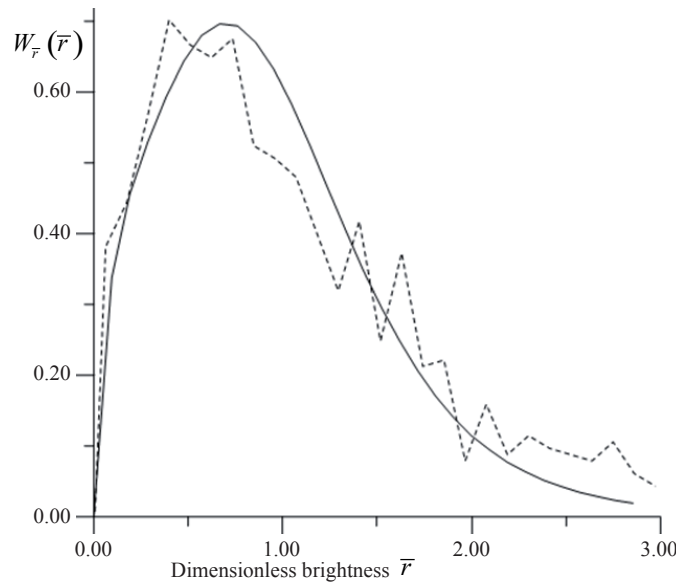


Fig. 7. Distribution density $W_r(\bar{r})$ of dimensionless brightness; *solid curve* — theoretical, *dashed curve* is obtained on the numerical experiments.

Рис. 7. Плотность распределения $W_r(\bar{r})$ безразмерной яркости; сплошная кривая — теоретическая, пунктирная кривая получена в результате численных экспериментов.

The statistical characteristics of SP of sea surface are very sensitive to changes in the reflecting surface $z = \zeta(x, y)$ and they can change in wide range, depending on the geometrical and physical characteristics of surface, namely, from degree and structure of the waves, presence of flow, internal waves and oil films on the surface. Therefore, the method for determining the statistical characteristics of the SP, presented above,

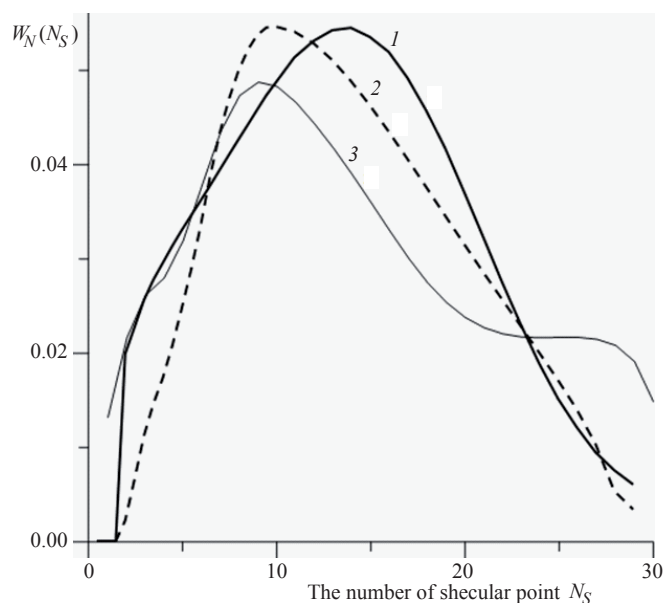


Fig. 8. Distribution density $W_N(N_S)$, of the number of SP, N_S ; curve 1 is obtained by the numerical experiment; curve 2 (dashed) is retrieved by the solution of the inverse problem using fractional values of N_S ; curve 3 is retrieved by the solution of the inverse problem using only integer values of N_S .

Рис. 8. Плотность распределения $W_N(N_S)$ числа зеркальных точек, N_S ; кривая 1 получена в результате численных экспериментов; кривая 2 (пунктирная) получена решением обратной проблемы используя дробные значения N_S ; кривая 3 получена решением обратной задачи с использованием только целых значений N_S .

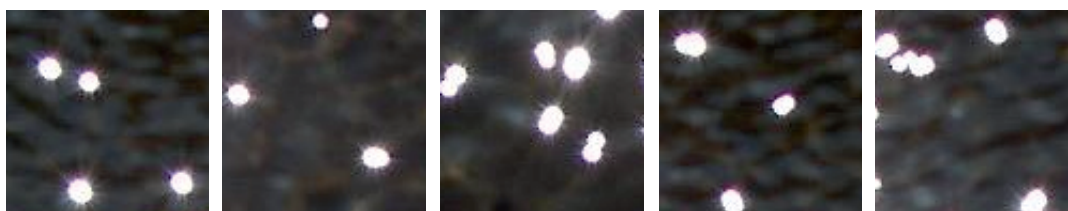


Fig. 9. Samples of the images of the Sun glitters on the waved basin, obtained by digital camera with the high time-spatial resolution.

Рис. 9. Примеры изображений солнечных бликов на взволнованной поверхности, полученных числовым фотоаппаратом с высоким пространственно-временным разрешением.

can be used for the remote sensing of the geometrical and physical characteristics of sea surface. For this it is necessary to investigate the interrelation between the geometrical and physical characteristics of surface and the statistical characteristics of SP — Sun glitters. As a result of these experimentally and theoretically studies the data base of the statistical characteristics of the Sun glitters can be created and used for determining sea surface wave state by solving of direct and invers problem. Moreover, from this parameters can be determined an instantaneous relief of the sea surface [13]. Consequently, new approach appears for solution of the series of problem, for example, the retrieving of instantaneous images of underwater objects, destroyed by waves; establishment of the laser connection between airborne vehicle and submarines and revealing of “dangerous harmonic” (tsunami).

Литература

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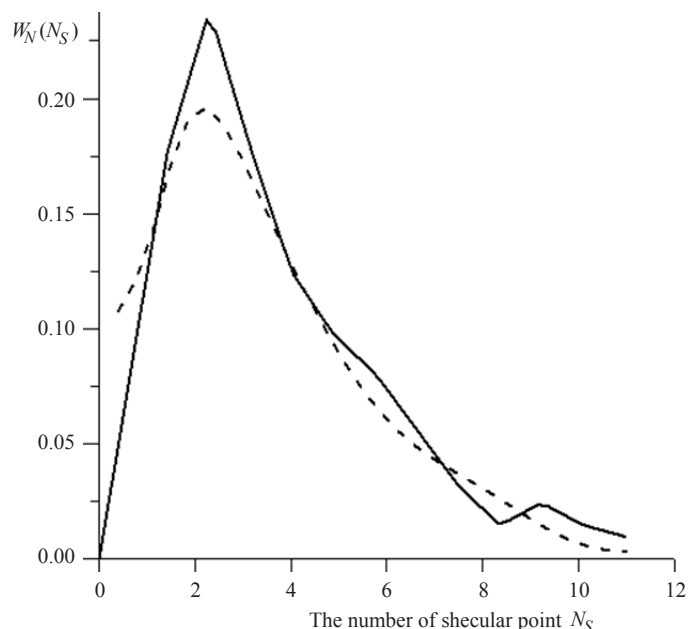


Fig. 10. Distribution density $W_N(N_S)$, of the number of SP, N_S : solid curve is obtained by processing the images of the Sun glitters; the dashed curve is retrieved by solving the inverse problem.

Рис. 10. Плотность распределения $W_N(N_S)$ числа зеркальных точек, N_S : сплошная кривая получена обработкой изображений солнечных бликов; пунктирная кривая получена решением обратной задачи.

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